How Important Are Sectoral Shocks?

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June 22, 2017

Abstract

I quantify the contribution of sectoral shocks to business cycle fluctuations in aggregate output. I develop and estimate a multi-industry general equilibrium model in which each industry employs the material and capital goods produced by other sectors. Using data on U.S. industries' input prices and input choices, I find that the goods produced by different industries are complements to one another as inputs in downstream industries' production functions. These complementarities indicate that industry-specific shocks are substantially more important than previously thought, accounting for at least half of aggregate volatility.

1 Introduction

What are the origins of business cycle fluctuations? Do idiosyncratic micro shocks—disturbances at individual firms or industries—have an important role in explaining short-run macroeconomic fluctuations? Or are shocks that prevail on all industries the predominant source?

I address these questions by constructing and estimating a multi-industry dynamic general equilibrium model in which both common and industry-specific shocks have the potential to contribute to aggregate output volatility. I find that sectoral shocks are important, accounting for considerably more than half of the variation in aggregate output growth.

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A challenge in identifying the relative importance of industry-specific shocks is that, because of input-output linkages, both aggregate and industry-specific shocks have similar implications for data on industries' sales. To see why, consider the following two scenarios. In the first, some underlying event (e.g., a surprise increase in the federal funds rate) reduces the demand faced by all industries, including the auto parts manufacturing, steel manufacturing (a supplier of auto parts), and auto assembly industries. In the second scenario, a strike occurs in the auto parts manufacturing industry, which temporarily reduces the demand faced by sheet-metal manufacturers, and increases the cost of establishments engaged in auto assembly. Even if industry-specific shocks are independent of one another, input-output linkages will induce co-movement in these industries' output and employment growth rates, just as in the first scenario. Intuitively, though, the more correlation across industry output growth that is observed, the more likely it is that common shocks are responsible for aggregate fluctuations.

But the extent to which industry activity co-moves depends on how easily consumers can substitute across the goods they consume, and how easily the firms within an industry can substitute across different factors of production. A particular amount of observed co-movement in output could result from one of two reasons. Production elasticities may be low, and common shocks relatively unimportant. Alternatively, elasticities of substitution may be large, and common shocks relatively important. The second challenge, then, emerges from the paucity of reliable, precise estimates of how easily industries can substitute across their inputs.[1]

In this paper, I confront these two related challenges sequentially. First, using data from the 1997 to 2013 BEA (Bureau of Economic Analysis) Annual Input-Output Tables, I estimate the relevant elasticities of substitution. In the data, the expenditure share of an industry on particular intermediate inputs are both volatile and positively correlated to the input's price. From these patterns, I estimate a relatively low value for the elasticity of substitution (which I call ε_M) among the intermediate inputs produced by different upstream industries: My point estimates of ε_M are consistently lower than 0.2, always significantly less than 1. In other words, different intermediate inputs are highly complementary to one another.

Second, armed with estimates of ε_M and the model's other salient elasticities of substitution, I construct a multi-industry dynamic general equilibrium model with which to infer industry productivity shocks. This model is an extension of that introduced in Foerster, Sarte, and Watson (2011), allowing for sectoral production functions that have non-unitary elasticities of substitution across inputs. Using the model, in conjunction with

[1] I discuss existing estimates of the relevant elasticities in Section 3.

data on industries output levels from 1960-2013, I back out the productivity shocks that each industry experienced over this sample period. I then extract the common component of these productivity shocks.

From here, I compute the fraction of the variation in aggregate output growth that can be explained by industry-specific (versus common) shocks. I find that most of the variation in aggregate output growth is attributable to the industry-specific components, 83 percent in my benchmark estimates. When I impose unitary elasticities of substitution on my model, I estimate that 21 percent of the variation in output volatility comes from industry-specific shocks. In sum, these results indicate that sectoral shocks are more important than previously thought, and that the difference is largely due to past papers' imposition of a unitary elasticity of substitution across different inputs in sectoral production functions. These results are robust to countries, industry classification schemes, treatment of trends, and other modeling choices.

This paper resolves the hypothesis, first advanced in Long and Plosser (1983), that independent industry-specific shocks generate patterns characteristic of modern business cycles. Models of business cycles typically portray fluctuations as the result of economy-wide, aggregate disturbances to production technologies and preferences. These disturbances, however, are difficult to justify independently, and may simply represent "a measure of our ignorance."[2] Given the results of the current paper, future research on the sources of business cycle fluctuations would benefit from moving beyond the predominant one-sector framework.

Related Literature: The current paper falls within the literature on multi-industry real business cycle models initiated by Long and Plosser (1983). Long and Plosser present a model in which the economy is composed of a collection of perfectly competitive industries. Each industry produces its output by employing a combination of labor and intermediate inputs. The intermediate input bundles of each industry are, in turn, combinations of goods that are purchased from other industries. Long and Plosser (1983) and others in this literature (e.g., Horvath 1998 and 2000; Dupor 1999; Acemoglu et al. 2012; and Acemoglu, Ozdaglar, and Tahbaz-Salehi 2017) use this framework to argue that idiosyncratic shocks to industries' productivities, by themselves, have the potential to generate substantial aggregate fluctuations.[3] These papers, however, do not allow for aggregate shocks; they are not attempting to assess the relative importance of industry-specific and aggregate shocks.

[2]This phrase was coined by Abramovitz (1956), when discussing the sources of long-run growth, but applies to our understanding of short-run aggregate fluctuations, as well. More recently, Summers (1986) and Cochrane (1994) have argued that it is *a priori* implausible that aggregate shocks can exist at the scale needed to engender the business cycle fluctuations that we observe.

[3]Among these papers, Dupor (1999) is exceptional. Instead of arguing that industry-specific shocks have the potential to produce business cycle fluctuations, he does the converse: He provides conditions on the input-output matrix under which industry-specific shocks are irrelevant.

Uniquely among the aforementioned papers, the model in Horvath (2000) accommodates non-unitary elasticities of substitution in consumers' preferences (across goods) and in the production of the intermediate input bundle (across inputs purchased from upstream industries).[4] His is the first article to articulate that lower elasticities of substitution "engender greater sectoral comovement... by reducing the ability of sectors to avoid the shocks of their input supplying sectors." (p. 83) A key difference between the current paper and Horvath (2000) is that the earlier paper does not attempt to estimate the values of these elasticities of substitution, nor does it seek to identify the role of common vs. sectoral shocks in generating aggregate fluctuations.

So, compared to the Long and Plosser literature, the current paper makes three advances. First, I extend Foerster, Sarte, and Watson (2011)'s identification scheme to accommodate flexible substitution patterns in consumers' preferences and industries' production technologies. Second, using data on industries' input usage and input prices, I estimate these production elasticities. Together, these two contributions are necessary to arrive at the paper's main result, that industry-specific shocks play a much larger role in generating aggregate volatility than previously believed. As a tertiary contribution, I make a sequence of smaller advances: I allow for consumption good durability, consider a dataset that covers the entire economy,[5] and examine data from several developed economies.[6]

Outline: In the remainder of the paper, I spell out the multi-sector real business cycle model (Section 2); estimate how easily industries can substitute across inputs (Section 3); apply these estimated elasticities to the real business cycle model to re-examine the relative importance of industry-specific shocks (Section 4); and conclude (Section 5). In Appendix A, I provide some additional details on the datasets used in the paper.[7]

[4] There are papers in other fields which focus more closely on these elasticities: Johnson (2014) and Boehm, Flaaen, and Pandalai-Nayar (2015a) study the transmission of shocks across international borders as a mechanism for generating cross-country co-movement and examine how the extent of model-predicted co-movement varies with production and preference elasticities.

[5] Foerster, Sarte, and Watson (2011) is unique in its application of the Federal Reserve Board's dataset on industrial production, a dataset that spans only the goods-producing sectors of the U.S. economy. Other papers (e.g., Long and Plosser 1983, Horvath 2000, and Ando 2014), employ datasets that cover the entire economy.

[6] A parallel literature attempts to gauge the relative importance of industry-specific shocks by estimating vector autoregressions (see Long and Plosser 1987, Stockman 1988, Shea 2002, and Holly and Petrella 2012). Yet another line of research constructs simple summary statistics of shocks to the largest firms or industries, relates these summary statistics to aggregate output movements, and in this way establishes the importance of micro shocks. Gabaix (2011) defines the *granular residual*—changes in productivity to the largest 100 firms—and shows that this statistic explains approximately one-third of the variation in GDP (see also di Giovanni, Levchenko, and Mejean 2014). Along these lines, Carvalho and Gabaix (2013) show that a summary statistic, one which measures the relative sizes of industries with different productivity volatilities, can help explain time-varying aggregate volatility.

[7] In the Online Appendices, I re-estimate one of the model's elasticities of substitution using plant-level data on manufacturers' input choices in Online Appendix B, describe the datasets from other countries (On-

2 Model

In this section, I present a multi-industry general equilibrium model. This is the simplest model that can be used to compare the importance of industry-specific and aggregate disturbances, and to estimate the production elasticities of substitution. The model is populated by a representative consumer and N perfectly competitive industries. I first describe the representative consumer's preferences, then the production technology of each industry, and finally the evolution of the exogenous variables.

2.1 Preferences

The consumer has balanced-growth-consistent preferences over leisure and the services provided by the N different consumption goods.[8]

The preferences of the consumer are given by the following utility function:

$$\mathcal{U} = \sum_{t=0}^{\infty} \beta^t \left[\log \left[\left(\sum_{J=1}^{N} \xi_J^{\frac{1}{\varepsilon_D}} \left(C_{tJ}\right)^{\frac{\varepsilon_D - 1}{\varepsilon_D}} \right)^{\frac{\varepsilon_D}{\varepsilon_D - 1}} \right] - \frac{\varepsilon_{LS}}{\varepsilon_{LS} + 1} \cdot \left(\sum_{J=1}^{N} L_{tJ} \right)^{\frac{\varepsilon_{LS}+1}{\varepsilon_{LS}}} \right] . \tag{1}$$

The demand parameters, ξ_J, reflect the time-invariant differences in the importance of industries' goods in the consumer's preferences; C_{tJ} equals the final consumption purchases on good/service J at time t. The elasticities of substitution parameterize how easily the representative consumer substitutes across the different consumption goods (ε_D) and how responsive the consumer's desired labor supply is to the prevailing wage (ε_{LS}).[9]

line Appendix C), report on a sequence of robustness checks (Online Appendices D and E), and characterize the solution of Section 2's model (Online Appendix F).

[8] In Online Appendix F, I extend the model to accomadate durability of certain consumption goods. This turns out to increase moderately the estimated importance of sectoral shocks for certain parameter configurations, and has no noticable effect for others; see Online Appendix E.

[9] Horvath (2000) and Kim and Kim (2006) use a more flexible specification regarding the disutility from supplying labor. In their specification, the second term in the period utility function is replaced by

$$-\frac{\varepsilon_{LS}}{\varepsilon_{LS} + 1} \cdot \left(\sum_{J=1}^{N} \left(L_{tJ}\right)^{\frac{\tau+1}{\tau}} \right)^{\frac{\tau}{\tau+1} \frac{\varepsilon_{LS}+1}{\varepsilon_{LS}}} .$$

The idea behind this specification is to "capture some degree of sector specificity to labor while not deviating from the representative consumer/worker assumption." Horvath (2000, p. 76) As it turns out, neither the volatility of aggregate economic activity nor the covariances of output across industries are particularly sensitive to the value of τ (see Table 9 of that paper). Moreover, since wages and hours are not among the observable variables that I am trying to match, the data that I employ in the following sections would have trouble identifying τ. For these reasons, I use the simpler specification of the disutility from labor supply.

2.2 Production and market clearing

Each industry produces a quantity (Q_{tJ}) of good J at date t using capital (K_{tJ}), labor (L_{tJ}), and intermediate inputs (M_{tJ}) according to the following constant-returns-to-scale production function:

$$Q_{tJ} = A_{tJ} \cdot \left[(1-\mu_J)^{\frac{1}{\varepsilon_Q}} \left(\left(\frac{K_{tJ}}{\alpha_J} \right)^{\alpha_J} \left(\frac{L_{tJ}}{1-\alpha_J} \right)^{1-\alpha_J} \right)^{\frac{\varepsilon_Q - 1}{\varepsilon_Q}} + (\mu_J)^{\frac{1}{\varepsilon_Q}} (M_{tJ})^{\frac{\varepsilon_Q - 1}{\varepsilon_Q}} \right]^{\frac{\varepsilon_Q}{\varepsilon_Q - 1}} . \quad (2)$$

The parameters μ_J and α_J reflect long-run averages in each industry's usage of intermediate inputs, labor, and capital. These parameters will eventually be inferred from the factor cost shares of each industry. A_{tJ} is the factor-neutral of industry J at time t. For now, these productivity terms can be correlated, across industries, in any arbitrary fashion. The parameter ε_Q dictates how easily factors of production are substituted.[10]

The evolution of capital, for each industry J, is given in Equation 3:

$$K_{t+1,J} = (1-\delta_K) K_{tJ} + X_{tJ} \ . \quad (3)$$

The capital stock is augmented via an industry-specific investment good, X_{tJ}, and depreciates at a rate δ_K that is common across industries. The industry-specific investment good is produced by combining the goods produced by other industries. The Γ^X_{IJ} indicate how important industry I is in the production of the industry J specific investment good, while ε_X parameterizes the substitutability of different inputs in the production of each industry's investment bundle:

$$X_{tJ} = \left(\sum_{I=1}^{N} \left(\Gamma^X_{IJ} \right)^{\frac{1}{\varepsilon_X}} \left(X_{t,I\to J} \right)^{\frac{\varepsilon_X - 1}{\varepsilon_X}} \right)^{\frac{\varepsilon_X}{\varepsilon_X - 1}} . \quad (4)$$

Analogously, the intermediate input bundle of industry J is produced through a combination of the goods purchased from other industries:

$$M_{tJ} = \left(\sum_{I=1}^{N} \left(\Gamma^M_{IJ} \right)^{\frac{1}{\varepsilon_M}} \left(M_{t,I\to J} \right)^{\frac{\varepsilon_M - 1}{\varepsilon_M}} \right)^{\frac{\varepsilon_M}{\varepsilon_M - 1}} . \quad (5)$$

In Equation 5, ε_M parameterizes the substitutability of different goods in the production

[10] In a robustness check (see column 2 of Table 4), I will consider labor-augmenting instead of TFP shocks. With ε_Q equal to 1, shocks to labor-augmenting productivity and TFP cannot be separately identified. With non-unitary elasticities of substitution, the paper's main results could *a priori* be sensitive to how the exogenous productivity term affects output.

of each industry's intermediate input bundle. The Γ^M_{IJ} indicate how important industry I is in the production of the industry J specific intermediate input.

To emphasize, the parameters Γ^M_{IJ}, Γ^X_{IJ}, α_J, and μ_J are time invariant. As such, movements in the share of J's expenditures spent on different factors of production are due, only, to the shocks to industries' productivity.

From the cost-minimization condition of the industry J representative firm, the relationship between the intermediate input cost share of industry J and the industry J specific intermediate input price (denoted P^{in}_{tJ}) is log-linear, with slope $1 - \varepsilon_Q$:[11]

$$\Delta \log \left(\frac{P^{in}_{tJ} \cdot M_{tJ}}{P_{tJ} \cdot Q_{tJ}} \right) = (1 - \varepsilon_Q) \cdot \Delta \log \left(\frac{P^{in}_{tJ}}{P_{tJ}} \right) + (\varepsilon_Q - 1) \cdot \Delta \log A_{tJ} \ . \tag{6}$$

A similar set of calculations yields the following relationship that describes changes in an industry's purchases of a specific intermediate input:

$$\Delta \log \left(\frac{P_{tI} M_{t,I \to J}}{P^{in}_{tJ} M_{tJ}} \right) = (1 - \varepsilon_M) \cdot \Delta \log \left(\frac{P_{tI}}{P^{in}_{tJ}} \right) \ . \tag{7}$$

When $\varepsilon_Q = \varepsilon_M = 1$, as assumed in previous papers, an industry's input cost shares are constant, independent of input prices, a prediction that I will show to be at odds with the data.

Finally, the market-clearing condition for each industry states that output can be used for consumption, as an intermediate input, or to increase one of the N capital stocks:

$$Q_{tI} = C_{tI} + \sum_{J=1}^{N} \left(M_{t,I \to J} + X_{t,I \to J} \right) \ . \tag{8}$$

[11] The equivalence between sales and costs in the denominator of the left-hand side of Equation 6 comes from the assumption that each industry is perfectly competitive, with a constant returns-to-scale production function.

To derive Equation 6, take first-order conditions of Equation 2 with respect to intermediate input purchases:

$$\begin{aligned} P^{in}_{tJ} &= P_{tJ} \cdot \frac{\partial Q_{tJ}}{\partial M_{tJ}} \\ &= P_{tJ} \cdot (A_{tJ})^{\frac{\varepsilon_Q - 1}{\varepsilon_Q}} (M_{tJ})^{-\frac{1}{\varepsilon_Q}} (\mu_J \cdot Q_{tJ})^{\frac{1}{\varepsilon_Q}} \ . \\ \left(P^{in}_{tJ}\right)^{\varepsilon_Q} &= (P_{tJ})^{\varepsilon_Q} (A_{tJ})^{\varepsilon_Q - 1} (M_{tJ})^{-1} \mu_J \cdot Q_{tJ} \ . \end{aligned}$$

Taking logs, re-arranging, and computing the difference across two adjacent periods gives Equation 6.

2.3 Evolution of the exogenous variables and the model filter

Using A_t to denote the vector of productivity levels in the N industries, $(A_{t1}, ..., A_{tN})'$, I specify the evolution of productivity as a geometric random walk:

$$\log A_{t+1} = \log A_t + \omega_{t+1}^A \ . \tag{9}$$

For now, the productivity shocks' covariance matrices are left unspecified. I will add some structure to these matrices in Section 4.

As in Foerster, Sarte, and Watson (2011), in a competitive equilibrium, the vector of industries' output growth rates admits a VARMA(1, 1) representation. Thus, the evolution of output can be written as:

$$\Delta \log Q_{t+1} = \Pi_1 \Delta \log Q_t + \Pi_2 \omega_t^A + \Pi_3 \omega_{t+1}^A \ . \tag{10}$$

The $N \times N$ matrices Π_1, Π_2, and Π_3 are functions of the model parameters. I solve for these matrices in Online Appendix F.

Solving Equation 10 for ω_{t+1}^A yields the filter:

$$\omega_{t+1}^A = (\Pi_3)^{-1} \Delta \log Q_{t+1} - (\Pi_3)^{-1} \Pi_1 \Delta \log Q_t - (\Pi_3)^{-1} \Pi_2 \omega_t^A \ . \tag{11}$$

With some initial condition for the productivity shock (e.g., $\omega_0^A = 0$), one could iteratively use data on sectoral growth rates to infer the productivity shocks at each point in time. I will apply this procedure in Section 4. But first, I must determine values for the model's elasticities of substitution. The example in the following subsection explains why.

2.4 Why do the elasticities matter?[12]

Before turning to the empirical analysis, I work through a special case of the model. This special case yields a relatively simple set of expressions for the relationship between the model parameters, the exogenous productivity shocks, and each industry's output. With this relationship in hand, I then discuss the intuition behind why imposing unitary elasticities of substitution may lead one to understate the role of industry-specific shocks.

Compared to the benchmark model, I make a number of simplifying assumptions. I assume that a) all goods depreciate fully each period; b) there is no physical capital in production; c) each industry has identical production functions; d) the consumer's preference

[12] This subsection is related to the technical appendix of Carvalho and Gabaix (2013). The main difference, besides assumptions (a)-(e), is that Carvalho and Gabaix impose that $\varepsilon_M = 1$ and allow for some adjustment costs to aggregate labor.

weight is the same for each of the N goods; and e) the input-output matrix has $\frac{1}{N}$ in each entry. Relaxing these assumptions would not overturn the example's main message, that higher elasticities of substitution generate less correlated output for a given set of correlations among the underlying productivity shocks.

The overall aim of the paper's model is to use data on industries' output to recover the degree to which productivity shocks are correlated across industries. If industry output data indicate that productivity shocks are correlated, then aggregate shocks will be assigned to play a primary role in generating industries' output (and, correspondingly, aggregate output) fluctuations. With this in mind, in Online Appendix F.6, I work out the following (log-linear, around the point at which $A_I = 1$ for all I) approximation for each industry's output as a function of the productivity in each industry:

$$\begin{aligned} \log Q_{tI} \quad \approx \quad & \log \frac{1}{N} + \frac{1}{1-\mu} \log \left(\frac{1}{1-\mu} \right) + \underbrace{\left(\mu \varepsilon_M + (1-\mu)\, \varepsilon_D \right)}_{①} \log A_{tI} \\ & + \frac{1}{N} \underbrace{\left[\left(\frac{1}{1-\mu} \right)^2 - \left(\mu \varepsilon_M + (1-\mu)\, \varepsilon_D \right) \right]}_{②} \sum_{J=1}^{N} \log A_{tJ} \end{aligned} \tag{12}$$

Equation 12 is helpful as it allows one to relate the covariance of industries' gross output as a function of industries' productivity shocks, and thus describes how one could recover the correlation of productivity shocks from data on gross output. The terms ① and ② in Equation 12 respectively specify the impact of industry-specific and common productivity changes on industry I's output level. Term ① is increasing in the two elasticities of substitution, ε_D and ε_M, and is minimized and equal to 0 when $\varepsilon_D = \varepsilon_M = 0$. In other words, regardless of the underlying correlation of the productivity terms, when production and preference elasticities are low, observed output will tend to strongly co-move. In contrast, in economies with larger production and preference elasticities, output will tend to co-move less for a given degree of correlation in the A terms.

In sum, the main takeaway from this simple example is that a given amount of observed output co-movement could arise either from low elasticities of substitution and correlated shocks or, alternatively, high elasticities of substitution and relatively uncorrelated shocks. So, to properly assess how important common versus independent shocks are, I must have reliable estimates parameterizing consumers' and firms' ease of substitution. This is the task to which I turn in the following section.

Before doing so, with the aim of providing the reader with some intuition, I briefly address a recurring question that I received while presenting this paper: Is the amplification of industry-specific shocks—where *amplification* is defined, here, as the aggregate output response following a shock in an individual sector—more severe with complementarities in production? On the one hand, when inputs are more complementary a (negative) productivity shock to a supplying industry (e.g., Steel) will lead to larger decreases in output for downstream industries (e.g., Motor Vehicles, Construction, etc...). On the other hand, the output decline in the industry experiencing the productivity shock will be smaller when its output is more complementary to the output of other industries. These two countervailing effects balance each other out in this simple example. Indeed, this must be the case, as the simple example of this subsection falls within the class of models studied in Hulten (1978) and Acemoglu et al. (2012). For this class of models, the aggregate impact of shocks to an individual sector is only a function of the sector's gross output share; to a first-order, the elasticities of substitution do not matter. Instead, the elasticities matter because they alter the way in which co-movement in fundamental shocks map to co-movement in observable data.

3 Estimates of the production elasticities

In this section, I estimate the model's key elasticities of substitution. With this goal in mind, I will apply industries' cost-minimization conditions, as given in Equations 6 and 7, to estimate ε_M and ε_Q. Recognizing the endogeneity of relative prices on the right-hand sides of these equations, I follow Shea (1993), Young (2014), and, especially, Acemoglu, Akcigit, and Kerr (2016) and use short-run industry-specific demand shifters as instruments. These shifts in demand arise from changes in military spending.

For this section, I use data from the BEA's GDP by Industry and Input-Output Accounts data spanning 1997 to 2013. The main variables that I construct from these tables are changes in i) industry J's output price index, $\Delta \log P_{tJ}$; ii) its intermediate input price index, $\Delta \log P^{in}_{tJ}$; iii) its intermediate input cost share, $\Delta \log \left(\frac{P^{in}_{tJ} M_{tJ}}{P_{tJ} Q_{tJ}}\right)$; and iv) the fraction of industry J's intermediate input cost shares that are due to purchases from industry I, $\Delta \log \left(\frac{P_{tI} M_{t,I\to J}}{P^{in}_{tJ} M_{tJ}}\right)$. So that I may combine production elasticity estimates with Dale Jorgenson's KLEMS data (which will be used in the following section), I collapse the 71 industries in the BEA data down to 30 industries. Appendix A contains a detailed description of the construction of the variables used in this section.

For four of the 30 industries, Figure 1 presents the relationship between $\Delta \log \left(\frac{P_{tI} M_{t,I\to J}}{P^{in}_{tJ} M_{tJ}}\right)$ and $\Delta \log \left(\frac{P_{tI}}{P^{in}_{tJ}}\right)$ for J's most important supplier industry. As an example, for the Furniture industry, which is depicted in the top-right panel, I plot the Furniture industry's intermediate input expenditure share of Lumber on the y-axis, and the price of Lumber relative to

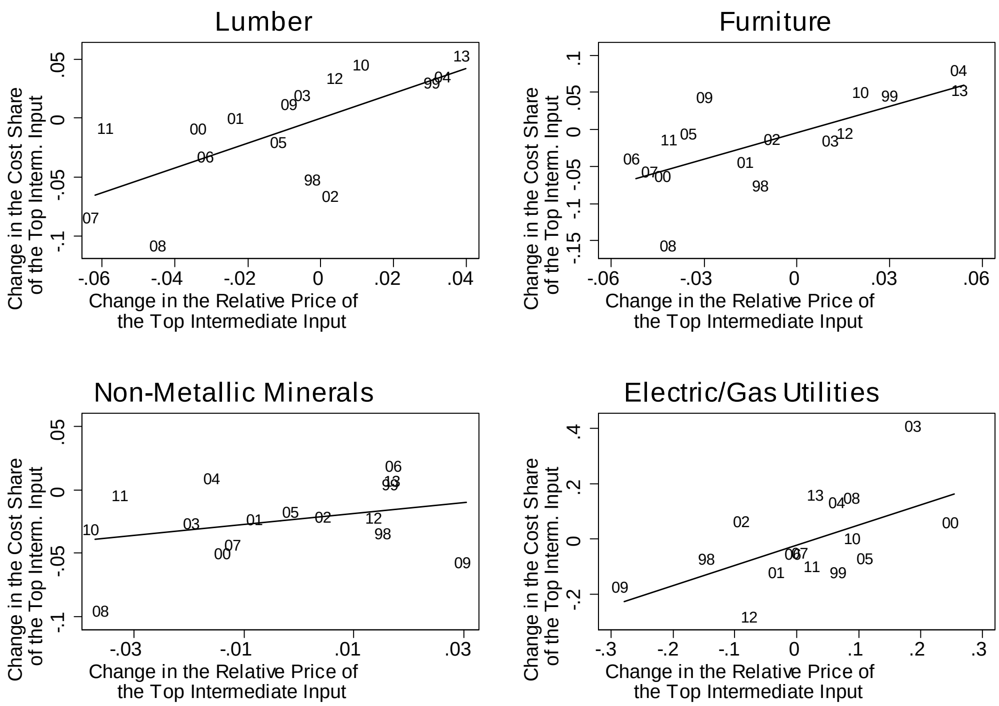

Figure 1: Relationship between changes in intermediate input purchases and intermediate input prices.

Notes: For each downstream industry, J, I take the most important (highest average intermediate input expenditure share) supplier industry, I. The x-axis of each panel gives $\Delta \log\left(\frac{P_{tI}}{P_{tJ}^{in}}\right)$. The y-axis gives, for each industry, changes in the fraction of industry J's intermediate input expenditures that go to industry I.

the price of Furniture's intermediate input bundle on the x-axis. The numbers on the plot give the last two digits of the year t. The main takeaway is that the share of a particular input among total intermediate input expenditures is positively correlated to the price of that input (relative to other intermediate inputs); this relationship is statistically significant for three out of the four industries (Non-metallic Minerals being the exception). Absent any omitted variables, Equation 7 would yield an unbiased estimate of $1 - \varepsilon_M$: The slope of $\Delta \log\left(\frac{P_{tI} M_{t,I\to J}}{P_{tJ}^{in} M_{tJ}}\right)$ on $\Delta \log\left(\frac{P_{tI}}{P_{tJ}^{in}}\right)$, averaging across the four plotted industries, is 0.85, which would yield an estimate of $\varepsilon_M = 0.15$.[13]

Similarly, ε_Q, the elasticity of substitution between intermediate inputs and value added, could be identified off of the slope of the relationship between changes in the intermediate input cost share, $\Delta \log\left(\frac{P_{tJ}^{in} M_{tJ}}{P_{tJ} Q_{tJ}}\right)$, and the relative price of intermediate inputs, $\Delta \log\left(\frac{P_{tJ}^{in}}{P_{tJ}}\right)$. Figure 2 plots this. All else equal, when ε_Q is less than 1, higher intermediate

[13]These four industries are broadly representative of those throughout the sample. In Online Appendix D, I depict this same relationship for all 30 industries.

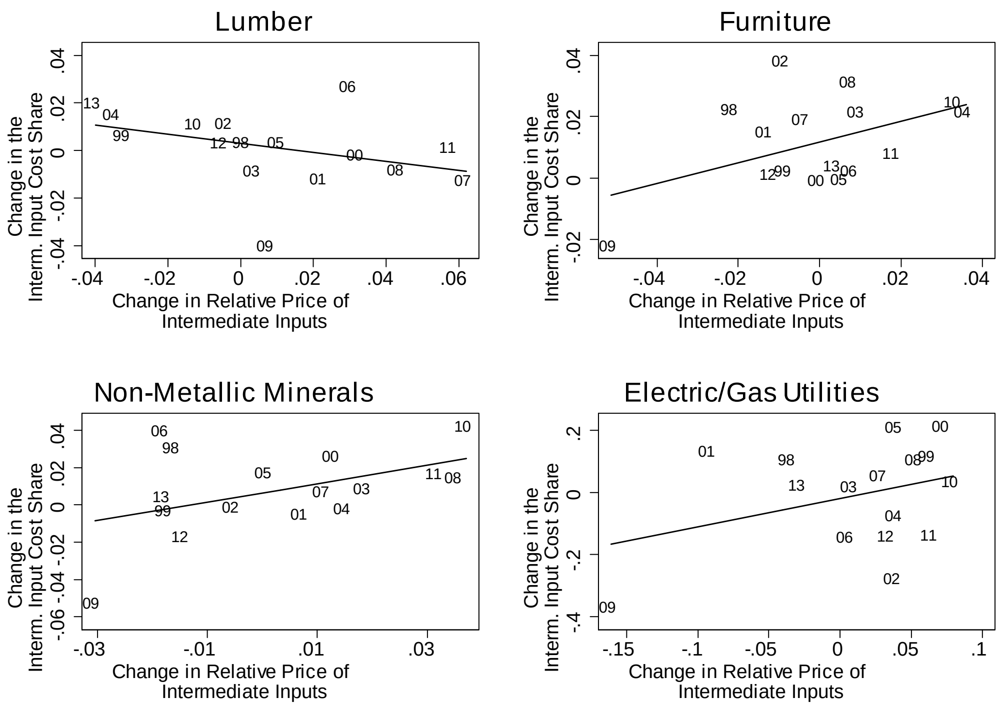

Figure 2: Relationship between changes in purchases of the intermediate input bundle and the relative price of the intermediate input bundle.
Notes: For each industry, J, I plot the relationship between changes in its cost share of intermediate inputs on the y-axis, and changes in the difference between the price of the intermediate input bundle and the marginal cost of production on the x-axis.

input prices are correlated with larger fractions of expenditures spent on intermediate inputs. For many, but certainly not all industries, this seems to be the case. The slope between $\Delta \log \left(\frac{P^{in}_{tJ} \cdot M_{tJ}}{P_{tJ} \cdot Q_{tJ}} \right)$ and $\Delta \log \left(\frac{P^{in}_{tJ}}{P_{tJ}} \right)$ is statistically distinct from zero for 12 of the 30 industries: negative for three of the commodity-related industries—Petroleum/Gas Extraction, Petroleum Refining, and Primary Metal Manufacturing—and positive for nine other industries. Overall, the slope of this line, for the average industry equals 0.4.

With the aim of more formally estimating ε_Q and ε_M, combine the cost-minimization conditions of each industry given in Equations 6 and 7:

$$\begin{aligned} \Delta \log \left(\frac{P_{tI} M_{t,I \to J}}{P_{tJ} Q_{tJ}} \right) &= \phi_t + (\varepsilon_M - 1) \left(\Delta \log P^{in}_{tJ} - \Delta \log P_{tI} \right) \\ &\quad + (\varepsilon_Q - 1) \left(\Delta \log P_{tJ} - \Delta \log P^{in}_{tJ} \right) + \eta_{t,IJ} \end{aligned} \tag{13}$$

Shifts in relative productivity, which are correlated with changes in relative prices and enter the error term in Equation 13, may lead to biased estimates of the production elasticities. According to the model presented in Section 2, shocks to industries' final demand would alter industries' demand for specific factors only through their effects on relative prices. I use a set of instruments from Acemoglu, Akcigit, and Kerr (2016) to capture these shifts in final demand.[14]

I define a set of three instruments, which exploit annual variation in military spending and heterogeneity in the extent to which different industries are suppliers to, either directly or indirectly, the military. They are defined as:

$$
\begin{aligned}
\text{military spending shock}_{tJ} &\equiv \sum_{I'} \text{Output\%}_{1997,J\to I'} \times \\
&\quad \mathcal{S}_{1997,I'\to\text{military}} \cdot \Delta \log\left(\text{Military Spending}_t\right), &(14)\\
\text{military spending shock}_{tI} &\equiv \sum_{I'} \text{Output\%}_{1997,I\to I'} \times \\
&\quad \mathcal{S}_{1997,I'\to\text{military}} \Delta \log\left(\text{Military Spending}_t\right), \quad \text{and} &(15)\\
\text{military spending shock}_{tJ\text{'s suppliers}} &\equiv \sum_{I} \frac{P_{tI} M_{t,I\to J}}{P^{in}_{tJ} M_{tJ}} \cdot \text{military spending shock}_{tI} \quad . &(16)
\end{aligned}
$$

With these three separate instruments, I aim to capture demand shifts which lead, respectively, to changes in P_{tJ}, P_{tI}, and P^{in}_{tJ} that are conditionally uncorrelated with $\eta_{t,IJ}$. In these equations, $\mathcal{S}_{1997,I\to\text{military}}$ is the share of industry I's output that is purchased by the "Federal National Defense" industries.[15] According to Equation 14, demand for an industry J's output will vary due to fluctuations in military spending either if it is a direct supplier to the military, if its main customers are important suppliers to the military, or if its main customers are indirect suppliers to the military. Among the industries in the sample, the "Other Transportation Industry"—in which ships, airplanes, and tanks are manufactured—has the highest $\mathcal{S}_{1997,I\to\text{military}}$. This industry has the strongest direct relationship with the military. Industries that are, on the other hand, indirectly reliant on purchases from the military include "Instruments" and "Petroleum Refining." Equation 15 is identical to Equation 14 except for the I subscript. And, to construct the military spending $\text{shock}_{tJ\text{'s suppliers}}$,

[14] The other demand shifter used by Acemoglu, Akcigit, and Kerr (2016) focuses on changes in industry demand resulting from China's consequential export expansion. Between 1995 and 2011, China's gross output exports to the United States, as a share of U.S. GDP, has increased from 0.5 percent to 2.7 percent. More importantly for the purposes of the current paper, growth in China's exports to the U.S. dramatically differ across industries. But, in the first-stage estimates of Equation 13, increased exports from China are associated with an *increase* in prices, counter to the motivation for the instrument.

[15] To define $\text{Output\%}_{1997,J\to I'}$, write $\mathcal{S}_{1997,J\to I'}$ as the share of industry J's output that is purchased by industry I' and store these elements in a matrix $\mathbf{S}$. Then, $\text{Output\%}_{1997,J\to I'}$ is the J,I' element of the matrix $\mathbf{I}+\mathbf{S}+\mathbf{S}^2+\mathbf{S}^3+...=(\mathbf{I}-\mathbf{S})^{-1}$.

Note that while Acemoglu, Akcigit, and Kerr (2016) motivate this definition using Cobb-Douglas sectoral production functions, the same definition—as a demand shifter—is compatible with CES production functions as well. See Online Appendix F.7.

I compute the average military spending shock of industries I, weighting each supplying industry by the extent to which they supplied intermediate inputs to industry J.

Table 1 presents the coefficient estimates from regressions defined by Equation 13. The first two columns present OLS estimates. In the IV specifications, given in the final two columns, the instruments have the expected relationship with the relative price variables: Increased demand from federal spending is positively related with the price of that industry's good. In these specifications, the instruments are sufficiently powerful to yield reliable, unbiased estimates of ε_M and ε_Q. For these specifications, the point estimates are actually slightly negative ε_M, around -0.1, though one cannot reject zero (or slightly positive values) for this elasticity of substitution. The right endpoint of a 90 percent confidence interval is approximately 0.2. For ε_Q, the OLS estimates result in an estimate of 1.2-1.3; the IV specifications produce estimates closer to 0.8-0.9. In these specifications, the standard errors for ε_Q are substantially larger: unit elasticities—as used previously in the literature—cannot be rejected.

In Online Appendix D, I report results from regressions which estimate the slopes of the relationships between input expenditures and prices for different countries; using different—either coarser (with 9 industries) or finer (with 67 industries)—industry classification schemes; using a longer definition of a time period; and specifications for which these slopes are separately estimated for different subsamples of industries. The results in the appendix accord with those presented in Table 1. Here, I summarize the results of these exercises. First, with more coarsely defined industries, the estimated elasticities are similar, but the instruments in the IV specifications are now weak. Second, estimates of ε_M and ε_Q are nearly identical to those in Table 1 with samples that include more upstream observations per downstream observation $\times$ year. Third, estimates of ε_M are somewhat larger, and estimates of ε_Q are somewhat smaller, with longer time periods (two years, instead of one year). Fourth, using the World Input Output Tables (WIOT), I estimate the slopes of intermediate input cost share versus intermediate input price relationships for a sample of six developed countries—Denmark, France, Italy, Japan, the Netherlands, and Spain. While, for these countries, I cannot apply variation in military spending as an instrument, the OLS estimates for these six countries are similar to those in the first two columns of Table 1: the estimates of ε_M are slightly larger (though still significantly smaller than 1), while estimates of ε_Q are somewhat smaller. In sum, the results from Table 1 are broadly, but not universally, robust to different samples and specifications. In all specifications, ε_M is safely well below 1.

While I am not aware of any previous research aimed at estimating ε_M, the estimates of ε_Q presented in Table 1 accord with the few existing estimates for this parameter.[16]

[16] Boehm, Flaaen, and Pandalai-Nayar (2015b) study the impact of the 2011 Tōhoku earthquake on the

Second stage regression results				
ε_M	-0.07	-0.13	-0.13	-0.11
	(0.04)	(0.04)	(0.19)	(0.20)
ε_Q	1.18	1.27	0.84	0.88
	(0.06)	(0.06)	(0.44)	(0.35)
First stage: Dependent variable is $\Delta \log P_{tJ}^{in} - \log P_{tI}$				
military spending shock$_{tI}$			-0.75	-0.70
			(0.06)	(0.06)
military spending shock$_{tJ\text{'s suppliers}}$			0.96	1.11
			(0.09)	(0.12)
military spending shock$_{tJ}$			-0.12	-0.14
			(0.07)	(0.06)
F-statistic			66.42	17.48
First stage: Dependent variable is $\Delta \log P_{tJ} - \log P_{tJ}^{in}$				
military spending shock$_{tI}$			-0.13	0.00
			(0.04)	(0.04)
military spending shock$_{tJ\text{'s suppliers}}$			-0.30	0.11
			(0.06)	(0.08)
military spending shock$_{tJ}$			0.38	0.35
			(0.04)	(0.04)
F-statistic			28.04	12.31
Cragg-Donald Statistic			27.04[i]	40.54[i]
Wu-Hausman test p-value			0.66	0.52
Year Fixed Effects	No	Yes	No	Yes
N	4800	4800	4592	4592

Table 1: Regression results related to Equation 13.

Notes: The overall sample includes pairs of industries J, and, for each industry J, the top ten supplying industries, I. In the third and fourth columns, the sample size is reduced because of the exclusion of the Government industry. In the row labeled "Cragg-Donald Statistic", an "i" indicates that the test for a weak instrument is rejected at the 10 percent threshold. Within this table, the "military spending shock$_{tJ}$" term, the "military spending shock$_{tI}$" term, and the "military spending shock$_{tJ\text{'s suppliers}}$" term are given by Equations 14, 15, and 16. These three military shock terms are meant to predict changes in the three price terms that appear on the right-hand side of Equation 13.

Rotemberg and Woodford (1996) estimate ε_Q by running a regression of manufacturing industries' intermediate input expenditure shares against the relative price of intermediate inputs, instrumenting the relative price of intermediate inputs using the price of crude oil. For industries within the manufacturing sector, Rotemberg and Woodford estimate a value of 0.7 for the elasticity of substitution between the capital-labor and the intermediate input bundles. More recent papers, using variation in the unit prices that individual plants pay for different factors, yield estimates of ε_Q in a similar range. Oberfield and Raval (2015) regress plants' intermediate input cost shares against the wages prevailing in their local labor markets, then combine this plant-level estimate with information on within-industry dispersion in plants' intermediate input intensities to build an industry-level estimate of ε_Q. Their estimates of ε_Q lie between 0.6 and 0.9. In Online Appendix B, I follow a similar strategy, exploiting spatial variation in materials prices instead of spatial variation in wages. I arrive at estimates of ε_Q in the 0.4 to 0.8 range.

The model's other elasticities of substitution, in particular ε_D and ε_{LS}, will turn out to play only a secondary role in determining the importance of aggregate fluctuations. For these parameters, I will choose a wide range, centered around values that have been estimated in previous papers. With respect to the estimate of ε_D, Herrendorf, Rogerson, and Valentenyi (2013) consider long-run changes in broad sectors' relative prices and final consumption expenditure shares. Their benchmark estimate of the preference elasticity of substitution between expenditures on agricultural products, manufactured goods, and services is 0.9.[17,18] Regarding ε_{LS}, an extensive literature has estimated the Frisch labor supply elasticity, with estimates varying between 0.5 and 3; see Prescott (2006) and Chetty

input purchases of U.S. multinational firms which had a presence in Japan. Using exogenous variation provided by the earthquake, Boehm, Flaaen, and Pandalai-Nayar (2015b) estimate a firm-level elasticity of substitution between capital/labor and intermediate inputs (their ζ) and a firm-level elasticity of substitution between intermediate inputs sourced from Japan and everywhere else (their ω). The former elasticity corresponds to a firm-level version of this paper's ε_Q; see Oberfield and Raval (2015) or the current paper's Online Appendix B for an explanation on how elasticities of substitution in firm-level and industry-level production functions can differ. The latter estimate is similar in spirit, but distinct from, a firm-level version of ε_M.

[17] With respect to an industry classification scheme closer to the one used in the current paper, Ngai and Pissarides (2007) argue that "the observed positive correlation between employment growth and relative price inflation across two-digit sectors" (p. 430) supports an estimate of ε_D that is less than 1. Also, Oberfield and Raval (2015) estimate a preference elasticity of between 0.8 and 1.1 across two-digit manufacturing industries.

[18] To emphasize, ε_D parameterizes how easily the consumer can substitute *across* coarsely-defined industries' products (for example, the elasticity of substitution between Motor Vehicles and Furniture, or between Apparel and Construction). Broda and Weinstein (2006) and Foster, Haltiwanger, and Syverson (2008), among others, estimate a much larger elasticity of substitution in consumers' preferences. These larger elasticities of substitution are estimated using *within-industry* variation, and characterize how easily consumers substitute between, for example, ready-mix concrete produced by two different plants, or between different varieties of red wine.

et al. (2011) for two syntheses of this literature.

To summarize, Table 1 suggests that a value for ε_M close to 0 and one for ε_Q that is close to but less than 1 faithfully describe industries' ability to substitute across inputs. In the following section, I will refer to $\varepsilon_M = 0.1$, $\varepsilon_Q = 1$, and $\varepsilon_D = 1$ as my benchmark set of parameter values.[19] However, since the standard errors of ε_Q are somewhat large, and since I have not even attempted to identify ε_X, ε_D, or ε_{LS}, it will be necessary to apply a range of values for these parameters. In the following section, I will compute the aggregate importance of sectoral shocks applying different reasonable combinations of ε_X, ε_Q, ε_M, and ε_D to Section 2's model, and compare this estimated contribution of sectoral shocks to a calibration in which ε_X, ε_Q, ε_M, and ε_D are all set equal to 1.

4 Estimates of the importance of sectoral shocks

This section contains the main results of the paper. In this section, I describe the calibration of certain parameters and the procedure with which I estimate the importance of common productivity shocks (Section 4.1); present the estimates of the importance of sectoral shocks for different values of the preference and production elasticities (Section 4.2); and examine the sensitivity of the benchmark results to changes in sample, industry definition, country, and other details of the estimation procedure (Section 4.3). I discuss additional robustness checks in Online Appendix E.

4.1 Calibration and estimation details

Besides the preference and production elasticities, the model filter requires data on industries' output at each point in time along with information on the long-run-average relationships across sectors. I discuss these two requirements in turn.

Regarding the data on industries' output, I combine Dale Jorgenson's 35-Sector KLEMS dataset (which spans the 1960 to 2005 period) with the output data from the BEA Industry Accounts (spanning 1997 to 2013) that were used in the previous section.[20] From

[19] It is true that there is some weak evidence in favor of $\varepsilon_Q < 1$. However, given the large variability of the estimates of ε_Q, I will choose the conventional value of 1 for ε_Q for the benchmark parameter configuration, and consider a secondary specification with $\varepsilon_Q = \frac{4}{5}$ in many of the other robustness checks.

The confidence intervals for ε_M in the final columns of Table 1 span both positive and negative values (in fact the point estimates from the IV regressions are negative.) Negative, statistically distinguishable from zero estimates of ε_M would be troublesome, as this would indicate that some component of the Section 2 model is mis-specified. This is not the case here, but the choice of $\varepsilon_M = 0.1$ does require some justification. I choose 0.1 as my benchmark value for ε_M as it lies in the middle of positive portion of the 90 percent confidence interval for this parameter.

[20] The Jorgenson data can be found on his home page; see http://scholar.harvard.edu/jorgenson/data .

these two datasets, I take information on industries' gross output, using industry-specific price deflators.

The parameters ξ_J, μ_J, α_J, Γ^X_{IJ}, and Γ^M_{IJ} are chosen to match the model-predicted cost shares to the corresponding values in the data. These parameters contain only information about the steady-state of the equilibrium allocation. The demand shares, ξ_J, are chosen so that the model's steady-state consumption choices are proportional to the amount that the industry sells to consumers or as government consumption expenditures; the ξ_J are restricted to sum to 1. The other parameters are chosen to match factor intensities, for each industry-factor pair. For instance, μ_J is the value that equates the model-predicted intermediate input cost share with the empirical counterpart.[21] The empirical values that are used to calibrate the factor intensities are described in Appendix A. Online Appendix F.1 provides additional details on the calibration of the parameters relevant to the steady state.[22]

I choose β and δ_K based on the values used in past analyses. I set the discount factor, β, to 0.96 and the capital depreciation rate, δ_K, to 0.10. I set the labor supply elasticity, ε_{LS}, equal to 2, and explore the sensitivity of the main results to this parameter in Table 5.

These calibrated parameters define the Π_1, Π_2, and Π_3 matrices that appear in Equation 11. This equation—which I reproduce for the reader's convenience below—can be used to infer each period's productivity shocks.

$$\omega^A_{t+1} = (\Pi_3)^{-1}\Delta \log Q_{t+1} - (\Pi_3)^{-1}\Pi_1 \Delta \log Q_t - (\Pi_3)^{-1}\Pi_2 \omega^A_t \; .$$

I apply two procedures to recover estimates of the ω^As. First, following the approach of Foerster, Sarte, and Watson (2011), I initialize the first-period productivity shocks at 0, $\omega_0 = 0$, and then iteratively apply Equation 11. This procedure is infeasible for certain sets of parameter values. For particular parameter configurations, some eigenvalues of $(\Pi_3)^{-1}\Pi_2$ are greater than 1 in absolute value. In this case, data on output changes alone cannot fully identify the productivity shocks.[23] A second issue arises, as some of the eigenvalues of Π_3 continuously pass from positive to negative values (or vice versa) as the chosen calibrated

Jorgenson, Gollop, and Fraumeni (1987) provide an extensive description of this dataset.

[21]When $\varepsilon_Q = 1$, the intermediate input cost share and μ_J are equal to one another. Alternatively, when intermediate inputs are gross complements or gross substitutes to other factors of production, the model-predicted cost share will also depend on the relative prices of the intermediate input bundle and the price of the other factors of production.

[22]In Online Appendix E, I examine the sensitivity of Section 4.2's results to using 1972, instead of 1997, as the year to which the steady-state allocation is calibrated.

[23]For parameter combinations for which at least one eigenvalue of $(\Pi_3)^{-1}\Pi_2$ is greater than 1 in absolute value, the "poor man's invertibility condition" in Fernández-Villaverde et al. (2007) is violated.

parameters are continuously modified.[24] As a result, the smallest eigenvalue of Π_3 is close to zero for certain combinations of the calibrated parameters. When either of these two issues arise, as a second approach, I treat the initial productivity shocks as an unknown state, and apply the Kalman filter, using the output data in each period to iteratively produce estimates of each date's productivity innovation. In the parameter configurations for which the largest eigenvalue of $(\Pi_3)^{-1}\Pi_2$ is less than 1, and the smallest eigenvalue of Π_3 is sufficiently large, the two approaches produce the same estimates of the productivity shocks.

4.2 Results

With the estimates of ω in hand, I present two measures of the importance of sectoral shocks in shaping aggregate volatility. To compute the first measure, I perform factor analysis to extract the (single) common component of the ω^As. Then, with the covariance matrices of the industry-specific and common productivity shocks in hand, I recover the model-implied covariance matrices for industries' value added that result only from sector-specific shocks (call this Σ^{ind}) or from both sector-specific and common shocks (call this Σ^{all}).[25] With $\bar{v}$ denoting the N-dimensioinal vector that contains each industry's value added share, the fraction of aggregate output volatility that is explained by the independent component of industries' productivity shocks is given by:

$$R^2\left(\text{sectoral shocks}\right) = \frac{\bar{v}'\Sigma^{\text{ind}}\bar{v}}{\bar{v}'\Sigma^{\text{all}}\bar{v}} \ . \tag{17}$$

The second measure of the relative importance of the common shocks is the average sample correlation of the productivity shocks,

$$\bar{\rho}\left(\omega\right) = \sum_{I=1}^{N}\sum_{J=1}^{N}\text{correlation}\left(\omega_i, \omega_j\right) \ . \tag{18}$$

These two measures were also used by Foerster, Sarte, and Watson (2011) to summarize the importance of sectoral shocks.

Figure 3 displays these two summary measures for different values of ε_M and ε_Q. According to the left panel of this figure, when ε_D, ε_M, and ε_Q are all equal to 1—as is the case in almost all previous analyses of multisector real business cycle models—sector-specific

[24] To give an example, when ε_D, ε_M, and ε_Q equal 0.15, 0.25, and 1, and applying all of the other choices described in this subsection, the smallest eigenvalue of Π_3 is 0.036. Then, decreasing ε_M from 0.25 to 0.2 yields a minimum eigenvalue of Π_3 equal to -0.006. For ε_M near 0.2, then, the model filter given by Equation 11 will yield unreliable estimates of the ω_t^A.

[25] Online Appendix F.5 explains the calculations behind Σ^{ind} and Σ^{all}.

shocks account for 21 percent of aggregate volatility.[26] For these same values of ε_D, ε_M, and ε_Q, the average correlation of the productivity shocks is 0.19.

A lower calibrated value for the elasticity of substitution among intermediate inputs yields estimates for industries' productivity shocks that are less correlated with one another. This relationship, which was the main takeaway of the simple example given in Section 2.4, is depicted in the left panel of Figure 3. With ε_M and ε_D as $\frac{1}{10}$ and 1, respectively, the filter results in productivity shocks that have an average correlation of 0.06. Put differently, the correlations among industries' output growth rates could arise either through productivity shocks that are relatively correlated and goods that have relatively high levels of substitutability, or through nearly independent productivity shocks and complementarity across the goods that industries produce.

With lower estimates of the correlation among productivity shocks, the common component of these shocks will account for a smaller fraction of aggregate volatility. Indeed, for $\varepsilon_D = 1$ and $\varepsilon_Q = 1$, more than half of aggregate volatility is due to industry-specific shocks so long as $\varepsilon_M \leq 0.2$; see the left panel of Figure 3. With our benchmark configuration—(ε_D, ε_M, ε_Q) equal to (1, $\frac{1}{10}$, 1)—83 percent of the variation of aggregate output is due to sectoral shocks. The right panel of this figure illustrates that R^2 (sectoral shocks) is relatively unresponsive to the chosen value of ε_Q. This, too, accords with the example in Section 2.4. With $\varepsilon_M = 1$ and $\varepsilon_D = 1$, the fraction of variation explained by industry-specific TFP shocks is between 13 and 30 percent for $\varepsilon_Q \in [0.15, 1.45]$. In sum, within the range of elasticities that I have estimated in Section 3, complementarities among intermediate inputs are important for assessing the role of aggregate fluctuations, but the elasticity of substitution between value added and intermediate inputs is not.

[26] Foerster, Sarte, and Watson (2011) also perform a factor analysis on industries' productivity shocks. They compute the fraction of industrial production growth that is due to the first two factors. The remaining variation can be considered equivalent to the industry-specific productivity shocks in the current paper. The two common factors explain 80 percent of the variation in overall industrial production growth in the first third of the sample (1972 to 1983) and 50 percent in the latter two-thirds (1984 to 2007). Averaging over these periods, sectoral shocks contribute roughly 40 percent of industrial production volatility.

There are a few potential explanations for why my figures may differ from those in Foerster, Sarte, and Watson (2011). One important difference is that the Foerster, Sarte, and Watson (2011) analysis is restricted to the goods-producing sectors of the economy, while I study the entire private economy; Ando (2014) explores the implications of this difference in coverage on the estimated contribution of industry-specific shocks. Other differences include a difference in sample period (1960 to 2013 in the current paper, compared to 1972 to 2008 in Foerster, Sarte, and Watson 2011), and period length (one quarter in Foerster, Sarte, and Watson 2011 versus one year, here). I show in the Online Appendix that excluding the Great Recession somewhat increases the assessed role of industry-specific shocks: When $\varepsilon_D = \varepsilon_M = \varepsilon_Q = 1$, R^2 (sectoral shocks) is 32 percent without the Great Recession, instead of 21 percent when the whole sample period is included. Decreasing the period length would—on the other hand, with $\varepsilon_M = 1$ and $\varepsilon_D = 1$—have little effect on the relative importance of sectoral shocks; see the fourth and fifth columns of Online Appendix Table 15.

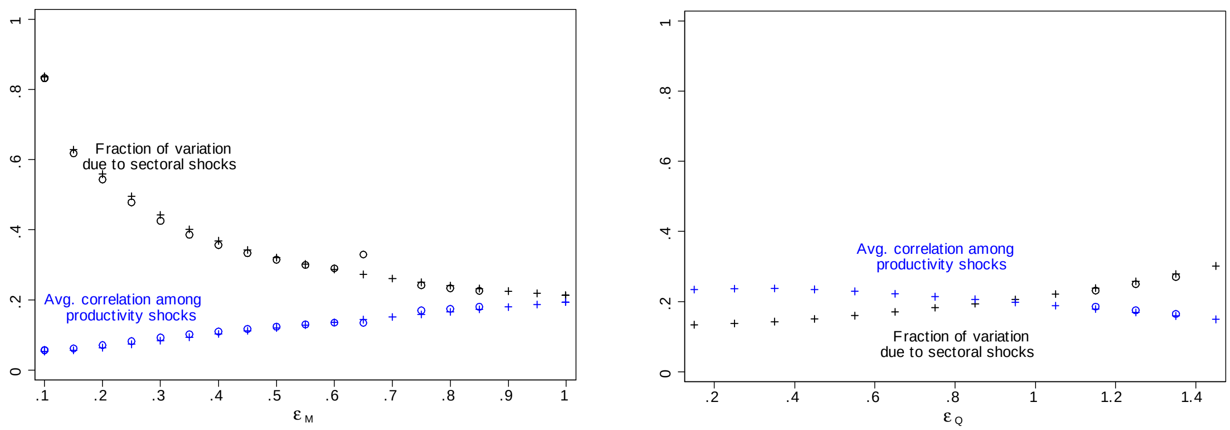

Figure 3: R^2(sectoral shocks) and $\bar{\rho}(\omega)$ for different values of ε_M and ε_Q .
Notes: In the left panel, $\varepsilon_Q = 1$; in the right panel, $\varepsilon_M = 1$; in both panels $\varepsilon_D = 1$. Hollow circles denote figures that result from the model filter, with ω_0 fixed at 0, iteratively applying Equation 11. "+" signs denote the figures that result from the Kalman filter.

Table 2 expands on these results. In this table, I compute the fraction of variation explained by sectoral shocks for different ε_D, ε_M, and ε_Q combinations. As in Figure 3, fixing a unit elasticity of substitution across intermediate inputs results in relatively high correlations among filtered productivity shocks, and a low estimated importance for industry-specific shocks. Even with improbably high values for ε_D, industry-specific shocks account for at least two-fifths of aggregate output volatility using Section 3's estimate of ε_M.

Next, I examine whether the choice of elasticities has implications for individual historical episodes. Figure 4 presents historical decompositions for two choices of ε_M. In both panels, $\varepsilon_D = \varepsilon_Q = 1$. In the left panel, I set $\varepsilon_M = 1$; and, in the right, $\varepsilon_M = \frac{1}{10}$. With relatively high elasticities of substitution across inputs, each and every recession between 1960 and the present day is explained almost exclusively by the common shocks. The sole partial exception is the relatively mild 2001 recession. In 2001 and 2002, Non-Electrical Machinery, Instruments, F.I.R.E. (Finance, Insurance, and Real Estate), and Electric/Gas Utilities—together accounting for GDP growth rates that were 2.0 percentage points below trend.

Table 3, along with the right panel of Figure 4, presents historical decompositions, now allowing for complementarities across intermediate inputs. Here, industry-specific shocks are a primary driver, accounting for a larger fraction of most, but certainly not all of, recent recessions and booms. According to the model-inferred productivity shocks, the 1974-75 and, especially, the early 1980s recessions were driven to a large extent by common shocks.[27] At the same time, the late 90s expansion and the 2008-09 recession are each

[27] While the common factor played a large role in the 1980s recessions, so too did the Motor Vehicles

$\varepsilon_M, \varepsilon_D, \varepsilon_Q$	R^2(sectoral shocks)	$\bar{\rho}(\omega)$
$1, 1, 1$	0.21^{kf}	0.19^{kf}
$1, 1, \frac{4}{5}$	0.19^{kf}	0.21^{kf}
$\frac{1}{10}, \frac{3}{5}, 1$	0.98^{kf}	0.04^{kf}
$\frac{1}{10}, \frac{4}{5}, 1$	0.99^{kf}	0.04^{kf}
$\frac{1}{10}, 1, 1$	0.83	0.06
$\frac{1}{10}, \frac{6}{5}, 1$	0.63	0.07
$\frac{1}{10}, \frac{7}{5}, 1$	0.56	0.08
$\frac{1}{10}, \frac{8}{5}, 1$	0.49	0.10
$\frac{1}{10}, \frac{9}{5}, 1$	0.43	0.11

Table 2: Robustness checks: R^2(sectoral shocks) and $\bar{\rho}(\omega)$ for different values of ε_D, ε_M, and ε_Q.
Notes: A "kf" indicates the use of the Kalman filter, as opposed to direct applications of Equation 11 to infer the ω productivity shocks.

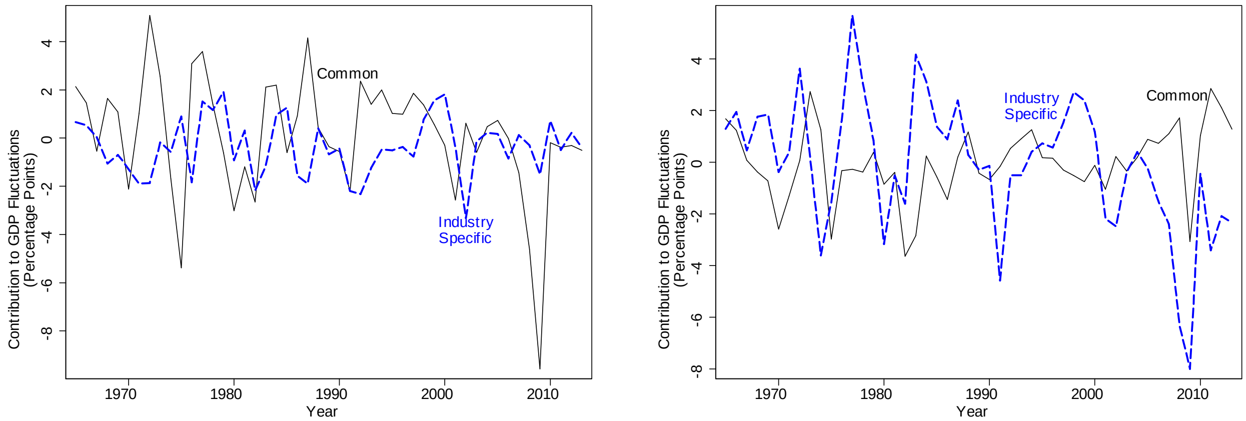

Figure 4: Historical decompositions.
Notes: The figure presents the percentage point change in each year's aggregate output (relative to trend) due to industry-specific and common shocks. In the left panel ε_D, ε_M, and ε_Q are all equal to 1. In the right panel, $\varepsilon_M = 0.1$, $\varepsilon_D = 1$, and $\varepsilon_Q = 1$.

1974-75		1980-82	
Other Services	-1.2%	Other Services	-1.4%
Construction	-1.0%	Construction	-0.7%
Government	0.4%	Motor Vehicles	-0.5%
Motor Vehicles	-0.3%	Warehousing	-0.4%
Warehousing	-0.3%	Wholesale & Retail	-0.3%
Common Factor	-1.7%	Common Factor	-4.9%
Total Change	-6.9%	Total Change	-10.2%
1996-2000		**2008-09**	
Other Services	1.7%	Other Services	-2.1%
Instruments	0.9%	Wholesale/Retail	-1.8%
F.I.R.E.	0.9%	F.I.R.E.	-1.1%
Construction	0.8%	Construction	-1.0%
Wholesale & Retail	0.4%	Motor Vehicles	-0.6%
Common Factor	-1.6%	Common Factor	-1.4%
Total Change	6.8%	Total Change	-15.7%

Table 3: Historical decompositions, using $\varepsilon_D = 1$, $\varepsilon_M = \frac{1}{10}$, and $\varepsilon_Q = 1$.
Notes: For four points in the sample, I report the five industries with the largest contributions to changes in aggregate ouput, the contribution of the common productivity shock, and the aggregate change in GDP, relative to trend.

more closely linked with industry-specific events. Instruments (essentially Computer and Electronic Products) and F.I.R.E. had an outsize role in the 1996-2000 expansion, while Wholesale/Retail, Construction, Motor Vehicles, and F.I.R.E. appear to have had a large role in the most recent recession. (Other Services, due to its large gross output share, appears as an important industry in most periods.) These model-inferred productivity shocks align with contemporaneous historical accounts.[28]

4.3 Sensitivity analysis

In Table 4, I examine the sensitivity of the assessed role of industry-specific shocks to the specification of productivity shocks, the industry classification scheme, and the calibration of industries' cost shares. In Online Appendix F, I specify and characterize a model with labor-augmenting productivity shocks. The first column reiterates the benchmark estimates

industry, especially in the first of the contractions: According to the model's historical decomposition, Motor Vehicles accounted for a 0.8 percent drop in aggregate output in 1979 and 1980.

[28] Related to the early 80s recession, Friedlaender, Winston, and Wang (1983) characterize the auto industry as "a state in flux. Not only has the Chrysler Corporation been perilously close to bankruptcy, but Ford and General Motors have suffered unprecedented losses in recent years." (pp. 1-2) Regarding the 1996-2000 expansion, Jorgenson and Stiroh (2000) analyze the role of information-technology-producing and consuming industries as a source of productivity acceleration during this period. And, finally, regarding the latest recession, Goolsbee and Krueger (2015) and Boldrin et al. (2016), respectively, chronicle distresses in Motor Vehicles and Construction.

with TFP shocks; the second column applies labor-augmenting productivity shocks. With $\varepsilon_Q < 1$, sectoral shocks contribute a larger fraction to aggregate volatility when productivity is assumed to be labor augmenting. In the third column, I establish that the results of Figure 3 are qualitatively robust to an nine-industry partition of the economy.[29] In the fourth column, I use data from 1972 (instead of 1997, as in the benchmark calculations) to infer the steady-state relevant parameters Γ^M_{IJ}, μ_J, α_J, and ξ_J.[30]

For the fifth column, in the data generating process, I replace factor-neutral productivity shocks in the government industry with government demand shocks (Online Appendix F.8 spells out the solution of the model filter with demand shocks; see in particular Equation 73). For all other industries, I retain TFP shocks instead of demand shocks. The rationale behind this robustness check stems from the application of military spending shocks as a source of identifying variation for ε_M and ε_Q in Section 3. Up to now, our model filter has precluded these types of shocks. So, the final column of Table 4 checks whether the misspecification which comes about because of the omission of military spending shocks is quantitatively important. It is not.

Table 5 presents the relative importance of sectoral shocks for various values of ε_X and ε_{LS}. In the specifications in which ε_M and ε_D both equal 1, industry-specific shocks contribute between 18 and 23 percent of aggregate volatility. In contrast, so long as $\varepsilon_M = \frac{1}{10}$, industry-specific shocks account for at least half of GDP volatility with $\varepsilon_D \in \left\{\frac{2}{3}, 1, \frac{4}{3}\right\}$.

As a third set of robustness checks, I examine the contribution of sectoral shocks to aggregate fluctuations in different countries. For this analysis, I employ data from the EU-KLEMS database, which describes industries' output growth rates for a range of developed countries between 1970 and 2007 (see Online Appendix C for a description of the dataset). As I estimate in Online Appendix D, industries' input choices and input prices, using the World Input Output Tables, suggest the elasticity of substitution among intermediate inputs may be higher for these six countries than are in Table 1, while the elasticity of substitution between intermediate inputs and value added may be lower. For this reason, in Table 6, I choose a somewhat higher value of ε_M, $\frac{1}{3}$ instead of $\frac{1}{10}$. As with the U.S. data, correlations among productivity shocks tend to be lower, and the assessed role of industry-specific shocks are higher, in specifications with lower values of the preference and production elastic-

[29] These industries are primary inputs (industries 1 to 3, according to Table 7), Construction (industry 4), Non-Durable Goods (industries 5 to 7 and 10 to 14), Durable Goods (industries 8, 9, and 15 to 23), Transport and Communications (industries 24 to 26), Wholesale and Retail (industry 27), F.I.R.E. (industry 28), Personal and Business Services (industry 29), and Government (industry 30). While it would be interesting to test the sensitivity of these results to a finer industry classification scheme, the necessary data are unavailable.

[30] The Capital Flows data necessary to construct Γ^X_{IJ} are unavailable for 1972. For this reason, I use the 1997 Capital Flows Table to infer the Γ^X_{IJ} for the robustness check corresponding to the penultimate column of Table 4.

	Benchmark	Labor-Aug. Productivity	9-Industry Classification	Use 1972 IO Table	Government Dem. Shocks
R^2 (sectoral shocks)					
$(\varepsilon_M, \varepsilon_D, \varepsilon_Q) = (1, 1, 1)$	0.21^{kf}	0.21^{kf}	0.26	0.18^{kf}	0.22^{kf}
$(\varepsilon_M, \varepsilon_D, \varepsilon_Q) = \left(1, 1, \frac{4}{5}\right)$	0.19^{kf}	0.22^{kf}	0.23	0.16^{kf}	0.20^{kf}
$(\varepsilon_M, \varepsilon_D, \varepsilon_Q) = \left(\frac{1}{10}, 1, \frac{4}{5}\right)$	0.81	0.93	0.85^{kf}	0.98	0.81
$(\varepsilon_M, \varepsilon_D, \varepsilon_Q) = \left(\frac{1}{10}, \frac{2}{3}, 1\right)$	0.99^{kf}	0.99^{kf}	0.95	0.98^{kf}	0.99^{kf}
$(\varepsilon_M, \varepsilon_D, \varepsilon_Q) = \left(\frac{1}{10}, 1, 1\right)$	0.83	0.83	0.82	1.00	0.83
$(\varepsilon_M, \varepsilon_D, \varepsilon_Q) = \left(\frac{1}{10}, \frac{4}{3}, 1\right)$	0.59	0.59	0.58	1.00	0.58
$\bar{\rho}(\omega)$					
$(\varepsilon_M, \varepsilon_D, \varepsilon_Q) = (1, 1, 1)$	0.19^{kf}	0.19^{kf}	0.26	0.17^{kf}	0.19^{kf}
$(\varepsilon_M, \varepsilon_D, \varepsilon_Q) = \left(1, 1, \frac{4}{5}\right)$	0.21^{kf}	0.19^{kf}	0.29	0.19^{kf}	0.21^{kf}
$(\varepsilon_M, \varepsilon_D, \varepsilon_Q) = \left(\frac{1}{10}, 1, \frac{4}{5}\right)$	0.06	0.07	0.15^{kf}	0.05	0.06
$(\varepsilon_M, \varepsilon_D, \varepsilon_Q) = \left(\frac{1}{10}, \frac{2}{3}, 1\right)$	0.04^{kf}	0.04^{kf}	0.11	0.01^{kf}	0.04^{kf}
$(\varepsilon_M, \varepsilon_D, \varepsilon_Q) = \left(\frac{1}{10}, 1, 1\right)$	0.06	0.06	0.13	0.08	0.06
$(\varepsilon_M, \varepsilon_D, \varepsilon_Q) = \left(\frac{1}{10}, \frac{4}{3}, 1\right)$	0.08	0.08	0.18	0.04	0.08

Table 4: Robustness checks: R^2(sectoral shocks) and $\bar{\rho}(\omega)$ for different values of ε_D, ε_M, and ε_Q.
Notes: A "kf" indicates the usage of the Kalman filter to infer the ω productivity shocks.

ε_X	1	1	1	1	$\frac{3}{5}$	$\frac{4}{5}$	$\frac{6}{5}$
ε_{LS}	2	$\frac{1}{2}$	1	4	2	2	2
R^2(sectoral shocks)							
$(\varepsilon_M, \varepsilon_D) = (1, 1)$	0.21^{kf}	0.18^{kf}	0.20^{kf}	0.23^{kf}	0.23^{kf}	0.22^{kf}	0.21^{kf}
$(\varepsilon_M, \varepsilon_D) = \left(\frac{1}{10}, \frac{2}{3}\right)$	0.99^{kf}	0.99^{kf}	0.99^{kf}	0.99^{kf}	0.96^{kf}	0.98^{kf}	1.00^{kf}
$(\varepsilon_M, \varepsilon_D) = \left(\frac{1}{10}, 1\right)$	0.83	0.78	0.81	0.92	0.97	0.86	0.81
$(\varepsilon_M, \varepsilon_D) = \left(\frac{1}{10}, \frac{4}{3}\right)$	0.59	0.51	0.55	0.62	0.65	0.62	0.56
$\bar{\rho}(\omega)$							
$(\varepsilon_M, \varepsilon_D) = (1, 1)$	0.19^{kf}	0.22^{kf}	0.21^{kf}	0.18^{kf}	0.18^{kf}	0.19^{kf}	0.20^{kf}
$(\varepsilon_M, \varepsilon_D) = \left(\frac{1}{10}, \frac{2}{3}\right)$	0.04^{kf}	0.04^{kf}	0.04^{kf}	0.04^{kf}	0.03^{kf}	0.04^{kf}	0.04^{kf}
$(\varepsilon_M, \varepsilon_D) = \left(\frac{1}{10}, 1\right)$	0.06	0.06	0.06	0.08	0.05	0.05	0.06
$(\varepsilon_M, \varepsilon_D) = \left(\frac{1}{10}, \frac{4}{3}\right)$	0.08	0.09	0.08	0.07	0.07	0.07	0.08

Table 5: Robustness checks: R^2(sectoral shocks) and $\bar{\rho}(\omega)$ for different values of ε_D and ε_M.
Notes: Throughout the table, $\varepsilon_Q = 1$. A "kf" indicates the usage of the Kalman filter, as opposed to direct applications of Equation 11, to infer the ω productivity shocks.

Country	Denmark	Spain	France	Italy	Japan	Netherlands
R^2 (sectoral shocks)						
$(\varepsilon_M, \varepsilon_D) = (1, 1)$	0.63^{kf}	0.87^{kf}	0.80	0.47	0.07	0.44
$(\varepsilon_M, \varepsilon_D) = \left(\frac{1}{3}, \frac{2}{3}\right)$	0.87	1.00	1.00	0.89	0.61	0.81
$(\varepsilon_M, \varepsilon_D) = \left(\frac{1}{3}, 1\right)$	0.80	0.98^{kf}	1.00	0.84	0.30	0.72
$(\varepsilon_M, \varepsilon_D) = \left(\frac{1}{3}, \frac{4}{3}\right)$	0.76^{kf}	0.96^{kf}	1.00	0.79	0.79	0.64
$\bar{\rho}(\omega)$						
$(\varepsilon_M, \varepsilon_D) = (1, 1)$	0.08^{kf}	0.08^{kf}	0.10	0.13	0.34	0.11
$(\varepsilon_M, \varepsilon_D) = \left(\frac{1}{3}, \frac{2}{3}\right)$	0.02	0.02	0.05	0.06	0.07	0.03
$(\varepsilon_M, \varepsilon_D) = \left(\frac{1}{3}, 1\right)$	0.02	0.04^{kf}	0.09	0.12	0.17	0.05
$(\varepsilon_M, \varepsilon_D) = \left(\frac{1}{3}, \frac{4}{3}\right)$	0.04^{kf}	0.05^{kf}	0.13	0.17	0.07	0.07

Table 6: Robustness checks: R^2(sectoral shocks) and $\bar{\rho}(\omega)$ for different values of ε_D and ε_M. Notes: Throughout the table, $\varepsilon_Q = 1$. A "kf" indicates the usage of the Kalman filter, as opposed to direct application of Equation 11, to infer the ω productivity shocks.

ities of substitution. For five of these six foreign countries, the sole exception being Japan, industry-specific productivity shocks account for at least half of aggregate volatility with $\varepsilon_M = \frac{1}{3}$.

In Online Appendix E, I demonstrate that the benchmark results are robust to i) the de-trending method, ii) the period length, iii) censoring outlier observations, iv) looking at different parts of the sample separately (excluding the Great Recession, or looking at the first half and second half of the sample separately), and v) modeling the durability of consumption goods.

5 Conclusion

In the short run, industries have limited ability to substitute across their inputs. This paper extends a standard multi-industry real business cycle model to explore the role of limited substitutability on the assessed role of sectoral shocks. A worked out example of this elaborate model indicates that observed relationships among industries' output growth rates could either be rationalized with high elasticities of substitution in production (or preferences) along with correlated shocks, or with low elasticities and uncorrelated shocks. Using data on industries' input choices and their input prices, I estimate that production elasticities of are, on balance, small. As a result, I find that sectoral shocks are more important than previously thought. Whereas previous assessments of multisector real business cycle models—based on unitary elasticities of substitution across inputs and consumption products—have concluded that industry-specific shocks account for less than half of aggregate volatility, the current paper indicates that sectoral shocks are the primary source of

GDP fluctuations.

A Details of the U.S. data

This section clarifies the sample construction and defines the variables used to estimate the model's elasticities of substitution. The main data sources are the 1997 to 2013 "Use" tables and the 1997 "Make" and Capital Flows tables, all from the Bureau of Economic Analysis; and Dale Jorgenson's KLEMS dataset.

Table 7 characterizes the way in which I classify industries. The NAICS codes refer to those in the Annual IO Tables. The third through fifth columns of Table 7 give the cost shares of capital, labor, and intermediate inputs. These are computed from the BEA GDP by Industry dataset. The intermediate input cost share is computed as the ratio of intermediate input expenditures relative to total gross output. The labor share is the ratio of labor compensation to total gross output. The remainder defines the capital cost share. The final column of Table 7 gives the consumption expenditure share of each industry. The consumption expenditures are taken from the BEA 1997 Input-Output Table, as sales to the following industry codes: F010 (personal consumption expenditures), F02R (residential private fixed investment), and F040 (exports). To compute consumption expenditures by the government sector, I combine F06C (Federal national defense: Consumption expenditures), F07C (Federal national nondefense: Consumption expenditures), and F10C (State and local: Consumption expenditures). With the aim of improving the numerical performance of the model filter, in my calibrations of ξ, I bound the preference weights, from below, at 0.006.

Section 3's analysis requires information on purchases across industries, the prices of each industry's good, and the prices of each industry's intermediate input bundle. For each year between 1997 and 2013, the Annual IO Tables contain information on the value of commodities that are used by different industries. The output price of each industry is taken from the BEA GDP by Industry dataset, using the Fisher ideal price index to aggregate up to the classification in Table 7. To compute each industry's intermediate input price, I follow a similar procedure: For each downstream industry, I require information on changes in its intermediate input bundle's price, for each year (this variable appears on the right-hand side of Equation 13). I use the Fisher ideal price index to compute change in the intermediate input prices:

$$\Delta \log P^{in}_{t+1,J} = \sum_{I=1}^{30} \frac{P_{tI} M_{t,I\to J} + P_{t+1,I} M_{t+1,I\to J}}{\sum_{I'=1}^{30} P_{tI'} M_{t,I'\to J} + P_{t+1,I'} M_{t+1,I'\to J}} \cdot \Delta \log P_{t+1,I},$$

#	Name	NAICS	Capital	Labor	Intermediate Inputs	Consumption
1	Agriculture, Forestry	11	0.32	0.10	0.58	0.008
2	Mining	212	0.23	0.25	0.52	0.001
3	Oil & Gas Extraction	211, 213	0.40	0.18	0.42	0.000
4	Construction	23	0.16	0.32	0.52	0.036
5	Food & Kindred Products	311, 312	0.14	0.12	0.74	0.043
6	Textile Mill Products	313, 314	0.08	0.23	0.69	0.003
7	Apparel, Leather	315, 316	0.09	0.22	0.69	0.017
8	Lumber	321	0.09	0.22	0.69	0.002
9	Furniture & Fixtures	337	0.13	0.31	0.56	0.004
10	Paper & Allied Products	322	0.16	0.21	0.63	0.003
11	Printing & Publishing	323, 511	0.18	0.29	0.53	0.009
12	Chemicals	325	0.27	0.16	0.58	0.020
13	Petroleum Refining	324	0.21	0.06	0.73	0.009
14	Rubber & Plastics	326	0.15	0.22	0.63	0.004
15	Non-Metallic Minerals	327	0.21	0.26	0.53	0.001
16	Primary Metals	331	0.09	0.19	0.71	0.002
17	Fabric. Metal Products	332	0.16	0.29	0.54	0.003
18	Non-Electrical Machinery	333	0.11	0.27	0.62	0.009
19	Electrical Machinery	335	0.18	0.24	0.58	0.005
20	Motor Vehicles	3361-3363	0.11	0.15	0.74	0.024
21	Other Transport. Equip.	3364-3369	0.10	0.30	0.60	0.008
22	Instruments	334	0.19	0.24	0.56	0.021
23	Misc. Manufacturing	339	0.21	0.32	0.47	0.009
24	Warehousing	48, 49	0.18	0.33	0.49	0.024
25	Communications	512, 513, 514	0.34	0.22	0.44	0.021
26	Electric/Gas Utilities	22	0.51	0.17	0.32	0.019
27	Wholesale & Retail	42, 44, 45	0.32	0.38	0.30	0.117
28	F.I.R.E.	52-53, HS, OR	0.51	0.15	0.33	0.170
29	Other Services	54-56, 60-89	0.18	0.43	0.38	0.250
30	Government	G	0.15	0.54	0.31	0.159

Table 7: Industry definitions, factor shares, and preference weights.

where, as in Section 2, $M_{t,I \to J}$ represents the physical units of intermediate inputs from industry I to industry J, and P_{tI} denotes the unit price of industry I's output. For each downstream industry, J, I compute the change in its intermediate input price—between years t and $t+1$—as the weighted average in the changes in the prices of the supplying industries, I, with the weights set at the year t and $t+1$ share of J's intermediate input purchases that come from industry I.

To construct Γ^M and Γ^K, I use data from the 1997 Input Output Table and Capital Flows Table. I make two adjustments to the 1997 Capital Flows Table when producing Γ^K. First, government investment is not measured in the Capital Flows Table. As a result, I need to apply information from the Input Output Table, which *does* contain sales to the government investment industry. These are measured as sales to the following industries: F06S, F06E, and F06N (Investment in Federal Defense); F07S, F07E, and F07N (Investment in Federal Nondefense); F10S, F10E, and F10N (Investment in State and Local Government). Second, ones needs to account for maintenance and repair expenditures, which are not included in the Capital Flows Table. As McGrattan and Schmitz (1999) report, maintenance expenditures are sizable, potentially accounting for 50 percent of total physical capital investment. Foerster, Sarte, and Watson (2011) use this finding as motivation for adding to the diagonal entries of Γ^K. I add a 35 percent share to the diagonal entries of Γ^K to account for these maintenance and repair expenditures. This augmentation presumes that capital-good repairs draw on within-industry resources (e.g., firms that produce a product use their own inputs to repair their capital equipment).[31]

For the robustness check on good durability, performed in Online Table 15, the set of durable goods are those designated as such by Basu, Fernald, and Kimball (2006), plus the Construction industry: Construction, Lumber, Furniture and Fixtures, Non-Metallic Minerals, Primary Metals, Fabricated Metal Products, Non-Electrical Machinery, Electrical Machinery, Motor Vehicles, Other Transportation Equipment, Instruments, and Miscellaneous Manufacturing.

References

Abramovitz, Moses. 1956. "Resource and Output Trends in the United States Since 1870." National Bureau of Economic Research, 1-23.

Acemoglu, Daron, Ufuk Akcigit, and William Kerr. 2016. "Networks and the Macroecon-

[31] There is a more practical rationale behind this alteration of the Capital Flows Table. When the diagonal entries of Γ^K are sufficiently low, several of the eigenvalues of $(\Pi_3)^{-1}\Pi_2$ are larger than 1 in absolute value, indicating that the calibrated models are non-invertible.

omy: An Empirical Exploration." *NBER Macroeconomics Annual*, 30(1): 273-335.

Acemoglu, Daron, Vasco M. Carvalho, Asuman Ozdaglar, and Alireza Tahbaz-Salehi. 2012. "The Network Origins of Aggregate Fluctuations." *Econometrica,* 80(5): 1977-2016.

Acemoglu, Daron, Asuman Ozdaglar, and Alireza Tahbaz-Salehi. 2017. "Microeconomic Origins of Macroeconomic Tail Risks." *American Economic Review*, 107(1): 54-108.

Ando, Sakai. 2014. "Measuring US Sectoral Shocks in the World Input-Output Network." *Economics Letters,* 125(2): 204-207.

Basu, Susanto, John G. Fernald, and Miles S. Kimball. 2006. "Are Technology Improvements Contractionary?" *American Economic Review*, 96(5): 1418-1448.

Boehm, Christoph, Aaron Flaaen, and Nitya Pandalai-Nayar. 2015a. "Complementarities in Multinational Production and Business Cycle Dynamics." mimeo.

Boehm, Christoph, Aaron Flaaen, and Nitya Pandalai-Nayar. 2015b. "Input Linkages and the Transmission of Shocks: Firm-Level Evidence from the 2011 Tōhoku Earthquake." mimeo.

Boldrin, Michele, Carlos Garriga, Adrian Peralta-Alva, and Juan M. Sanchez. 2016. "Reconstructing the Great Recession." mimeo.

Broda, Christian, and David E. Weinstein. 2006. "Globalization and the Gains from Variety." *Quarterly Journal of Economics*, 121(2): 541-585.

Carvalho, Vasco M., and Xavier Gabaix. 2013. "The Great Diversification and Its Undoing." *American Economic Review*, 103(5): 1697-1727.

Chetty, Raj, Adam Guren, Day Manoli, and Andrea Weber. 2011. "Are Micro and Macro Labor Supply Elasticities Consistent? A Review of Evidence on the Intensive and Extensive Margins." *American Economic Review*, 101(3): 471-475.

Cochrane, John. 1994. "Shocks." *Carnegie–Rochester Conference Series on Public Policy*, 41: 295-364.

di Giovanni, Julian, Andrei Levchenko, and Isabelle Mejean. 2014. "Firms, Destinations, and Aggregate Fluctuations." *Econometrica,* 82(4): 1303-1340.

Dupor, Bill. 1999. "Aggregation and Irrelevance in Multi-Sector Models." *Journal of Monetary Economics*, 43(2): 391-409.

Fernández-Villaverde, Jesús, Juan F. Rubio Ramírez, Thomas Sargent, and Mark Watson. 2007. "ABCs (and Ds) of Understanding VARs." *American Economic Review,* 97(3): 1021-1026.

Foerster, Andrew, Pierre-Daniel Sarte, and Mark Watson. 2011. "Sectoral vs. Aggregate Shocks: A Structural Factor Analysis of Industrial Production." *Journal of Political Economy*, 119(1): 1-38.

Foster, Lucia, John Haltiwanger, and Chad Syverson. 2008. "Reallocation, Firm Turnover, and Efficiency: Selection on Productivity or Profitability?" *American Economic Review*, 98(1): 394-425.

Friedlaender, Ann F., Clifford Winston and Kung Wang. 1983. "Costs, Technology, and Productivity in the U.S. Automobile Industry." *Bell Journal of Economics*, 14(1): 1-20.

Gabaix, Xavier. 2011. "The Granular Origins of Aggregate Fluctuations." *Econometrica*, 79(3): 733-772.

Goolsbee, Austan D., and Alan B. Krueger. 2015. "A Retrospective Look at Rescuing and Restructuring General Motors and Chrysler." *Journal of Economic Perspectives*, 29(2): 3-24.

Herrendorf, Berthold, Richard Rogerson, and Ákos Valentinyi. 2013. "Two Perspectives on Preferences and Structural Transformation." *American Economic Review*, 103(7): 2752-2789.

Holly, Sean, and Ivan Petrella. 2012. "Factor Demand Linkages, Technology Shocks, and the Business Cycle." *Review of Economics and Statistics*, 94(4): 948-963.

Horvath, Michael. 1998. "Cyclicality and Sectoral Linkages: Aggregate Fluctuations from Independent Sectoral Shocks." *Review of Economic Dynamics*, 1(4): 781-808.

Horvath, Michael. 2000. "Sectoral Shocks and Aggregate Fluctuations." *Journal of Monetary Economics,* 45(1): 69-106.

Hulten, Charles R. 1978. "Growth Accounting with Intermediate Inputs:" *Review of Economic Studies*, 45(3): 511-518.

Johnson, Robert. 2014. "Trade in Intermediate Inputs and Business Cycle Co-movement." *American Economic Journal: Macroeconomics,* 6(4): 39-83.

Jorgenson, Dale W., Frank Gollop, and Barbara Fraumeni. 1987. *Productivity and U.S. Economic Growth*, Harvard University Press.

Jorgenson, Dale W., and Kevin Stiroh. 2000. "Raising the Speed Limit: U.S. Economic Growth in the Information Age." *Brookings Papers on Economic Activity*, 2000(1): 125–235.

Kim, Kunhong, and Young Sik Kim. 2006. "How Important Is the Intermediate Input Channel in Explaining Sectoral Employment Co-movement Over the Business Cycle?" *Review of Economic Dynamics*, 9(4): 659-682.

Long, John B., and Charles I. Plosser. 1983. "Real Business Cycles." *Journal of Political Economy*, 91(1): 39-69.

Long, John B., and Charles I. Plosser. 1987. "Sectoral vs. Aggregate Shocks In The Business Cycle." *American Economic Review*, 77(2): 333-336.

McGrattan, Ellen and James Schmitz. 1999. "Maintenance and Repair: Too Big to Ignore."

Federal Reserve Bank of Minneapolis, Quarterly Review, 23: 2-13.

Ngai, L. Rachel, and Christopher A. Pissarides. 2007. "Structural Change in a Multisector Model of Growth." *American Economic Review*, 97(1): 429-443.

Oberfield, Ezra, and Devesh Raval. 2015. "Micro Data and the Macro Elasticity of Substitution." mimeo.

Prescott, Edward C. 2006. "Nobel Lecture: The Transformation of Macroeconomic Policy and Research." *Journal of Political Economy*, 114(2): 203-235.

Rotemberg, Julio J., and Michael Woodford. 1996. "Imperfect Competition and the Effects of Energy Price Increases on Economic Activity." *Journal of Money, Credit, and Banking*, 28(4): 549-577.

Shea, John. 1993. "The Input-Output Approach to Instrument Selection." *Journal of Business & Economic and Statistics*, 11(2): 145-155.

Shea, John. 2002. "Complementarities and Co-movements." *Journal of Money, Credit, and Banking*, 34(2): 412-433.

Stockman, Alan C. 1988. "Sectoral and National Aggregate Disturbances to Industrial Output in Seven European Countries." *Journal of Monetary Economics*, 21(2): 387-409.

Summers, Lawrence. 1986. "Some Skeptical Observations on Real Business Cycle Theory." *Federal Reserve Bank of Minneapolis, Quarterly Review*, 10: 23-27.

Young, Alwyn. 2014. "Structural Transformation, the Mismeasurement of Productivity Growth, and the Cost Disease of Services." *American Economic Review*, 104(11): 3635-3667.

Online Appendix to "How Important Are Sectoral Shocks?"

By Enghin Atalay

B Cross-sectional estimates

In this section, I will apply plant-level input price variation and materials usage to provide an alternate set of estimates of ε_Q. To do so, I pursue the following two-part strategy. For each industry, I estimate how easily individual plants substitute across their factors of production, by relating plants' materials purchases to their materials prices. Then, I apply the methods developed in Oberfield and Raval (2015), which allow me to combine information on a) the plant-level elasticity of substitution, b) the dispersion of materials cost shares, and c) the elasticity of plant scale to marginal costs so that I can ascertain the corroborating estimates of ε_Q.

To preview the main results of this section, the elasticity of substitution for the plant-level production function is approximately 0.65. Because within-industry variation in materials expenditure shares is small for each of the ten industries, the industry level production function's elasticity of substitution is only somewhat higher, 0.75. Moreover, across the industries in the sample, the industry-level elasticities of substitution are similar to one another.

B.1 Data source and sample

The data source, for this section, is the Census of Manufacturers. This dataset contains plant-level information for each manufacturer in the United States, and is collected once every five years, in years ending in a "2" or a "7." For certain industries, plants with greater than five employees are asked to provide information on each of the material inputs that they consume and each of the products that they produce. Critically, for the empirical analysis of this section, the Census Bureau elicits information on both the quantities and values of these inputs and outputs, allowing me to construct plant-level prices. Additionally, the Census Bureau records a plant identifier, which will allow me to compare the intermediate input purchases of the same plant across different time periods.

The sample in this section is identical to that which was used in an earlier paper (see Atalay 2014). The industries are those for which outputs and inputs are relatively homogeneous. This choice reflects a desire to, as much as possible, rule out heterogeneous quality as a source of input or output price variation. The ten industries that comprise the sample are

Sample	Units of Output	Material Inputs	N
Boxes, Year≤1987	Short Tons	Paper/Paperboard (90%)	1820
Boxes, Year≥1992	Square Feet	Paper/Paperboard (89%)	646
Ground Coffee	1000 Pounds	Green Coffee Beans (80%)	300
Ready-Mix Concrete	1000 Cubic Yards	Cement (53%), Sand/Gravel (28%)	3708
White Wheat Flour	50-Pound Sacks	Wheat (90%)	503
Gasoline	1000 Barrels	Crude Petroleum (84%)	692
Milk, Bulk	1000 Pounds	Unprocessed Whole Milk (88%)	127
Milk, Packaged	1000 Quarts	Unprocessed Whole Milk (72%)	2099
Raw Cane Sugar	Short Tons	Sugar Cane (93%)	177
Carded Cotton Yarn	1000 Pounds	Cotton Fibers (80%), Polyester Tow (10%)	431
Pooled	-	-	10,503

Table 8: Description of the 10 industries in the sample.
Notes: This table is a duplicate of Table 1 of Atalay (2014). The percentages that appear in the Material Inputs column are the fraction of materials expenditures that go to each particular material input. The Material Inputs column shows the inputs that represent greater than 6% of the average plant's total material purchases.

corrugated boxes (with the years 1972-1987 and 1992-1997 analyzed separately. The way in which material inputs are coded, for this industry, differs in the two parts of the sample), ground coffee, ready-mix concrete, white wheat flour, gasoline, bulk milk, packaged milk, raw cane sugar, and grey cotton yarn; see Table 8. For additional details regarding the sample, see Appendix B of Atalay (2014).

B.2 Environment and assumptions

Each industry, I, comprises a set of plants $i \in I$, who combine capital, labor, material inputs, and purchased services to produce a single product. The production function is constant-returns to scale; separable between material inputs, N, and other inputs, O; with constant elasticity of substitution, η_P:

$$\begin{aligned} Q_{it}(K_{it}, L_{it}, S_{it}, N_{it}) &= \left[(A_{it} \cdot O_{it})^{\frac{\eta_P - 1}{\eta_P}} + (B_{it} \cdot N_{it})^{\frac{\eta_P - 1}{\eta_P}}\right]^{\frac{\eta_P}{\eta_P - 1}}, \\ \text{where } O_{it} &= F(K_{it}, L_{it}, S_{it}) \end{aligned} \tag{19}$$

Also by assumption, F exhibits constant returns to scale. Plants are allowed to flexibly alter their input choices, including capital, each period. Furthermore, the factor prices that each plant faces, both for the material input and for the other input aggregate, are constant in the amount purchased. These assumptions serve a dual purpose. Not only do these assumptions greatly simplify the estimation of η_P, they also allow me to apply Oberfield and Raval (2015)'s methodology to estimate ε_Q from η_P.

Use P_{it}^{oth} and P_{it}^{mat} to denote the factor prices for a unit of the other input aggregate and the material input, respectively. Let A_{it} and B_{it} represent the two plant-level productivity measures (other-input-augmenting and materials augmenting).

The demand curve faced by each plant, i, has constant elasticity, ε_D:

$$Q_{it} = \exp\{\theta_{it}\} \cdot (P_{it})^{-\eta_D} \tag{20}$$

In Equation 20, θ_{it} represents a plant-year specific demand shifter. The assumption of a constant elasticity demand curve, while probably counterfactual, is again useful for multiple reasons. The constant-demand-elasticity assumption allows me to directly apply the Foster, Haltiwanger, and Syverson (2008) methodology to estimate η_D. Moreover, the same assumption is invoked by Oberfield and Raval (2015)—whose work I apply, here—in their aggregation of plant-level to industry level production functions.

The profit-maximizing levels of N_{it} and O_{it} yield the following expression for the material-output ratio:

$$\log\left[\frac{N_{it}}{Q_{it}}\right] = -\eta_P \cdot \log\left[\frac{P_{it}^{mat}}{P_{it}}\right] + \eta_P \cdot \log\left[\frac{\eta_D - 1}{\eta_D}\right] + (\eta_P - 1)\log B_{it} \tag{21}$$

This equation will form the basis of the estimation of η_P, a task to which I now turn.

B.3 The micro elasticity of substitution

In this subsection, I estimate the plant-level elasticity of substitution between purchased inputs and other inputs. The baseline regression that I run is:

$$n_{it} - q_{it} = -\eta_P \cdot \left(p_{it}^{mat} - p_{it}\right) + \epsilon_{it}\ . \tag{22}$$

In Equation 22, and throughout the remainder of the section, I use lower-case letters to denote the logged, de-meaned values of the variable of interest. In other words, η_P is estimated only using within industry-year variation. To emphasize, both n_{it} and q_{it} refer to the number of physical units, and not the values, of the material good that plant i purchases and the output that it produces.

Ordinary least squares results are presented in the first column of Table 9. For most industries, the estimate of η_P lies between 0.5 and 0.7, with concrete and flour having two of the lower estimates and bulk milk and raw cane sugar with two of the higher estimates.

There are at least two concerns regarding the interpretation of η_P—from an OLS estimate of Equation 22—as an estimate of the micro elasticity of substitution. First, to the extent that the constant elasticity of demand assumption—embodied in Equation 20—is violated, Equation 22 suffers from omitted variable bias. A positive correlation between $\log\left[\frac{\eta_D - 1}{\eta_D}\right]$ and $(p_{it}^{mat} - p_{it})$ will engender a positive bias in η_P. Second, I have assumed that the materials supply curve that each i faces is flat. It is likely, however, that each plant's factor supply curve is upward sloping. This instance of simultaneity bias—whereby a high-B_{it} plant pays a high materials price—will also engender a positive bias in η_P

I offer two different approaches to circumvent these problems. Fist, I append plant-level fixed effects to Equation 22. These fixed effects aim to capture long-run cross-sectional variation in the conditions in output and factor markets. As Foster, Haltiwanger, and Syverson (2008, 2016) argue, the factor market conditions that a plant faces are substantially more persistent than its productivity.

In a second specification, I instrument plants' output and materials prices with the prices paid and charged by competitor plants. Specifically, the two instrumental variables, for $p_{it}^{mat} - p_{it}$, are a) the year-t average materials price for plants that are within 50 miles of plant i, and b) the year-t average output price for plants that are within 50 miles of plant i. The idea behind these instruments is that the price of materials in nearby markets is correlated with the price that i pays for its material inputs (if, for example, there is spatial correlation in the abundance of primary inputs used in the production of i's intermediate inputs, or if there is a very productive, low marginal-cost supplier nearby), but should not in any other way affect the propensity for i have exceptionally high or low materials expenditure shares.[32]

Results from the two sets of regressions are given in the second and third columns of Table 9. In the second column, estimates of η_P range from 0.40 to 0.92, with the two largest estimates corresponding to two of the smaller-sample industries, coffee and sugar. The pooled estimate of η_P is 0.68.

The instrumental variables are weak for the six smallest samples. For this reason, the IV specification is performed only on the samples of plants in the corrugated boxes, ready-mix concrete, packaged milk, and petroleum industries. In the third specification, the parameter estimates are smaller and much less precisely estimated. The biggest difference is for the ready-mix concrete industry, for which the estimate of η_P is essentially 0.

[32] Results from first-stage regressions indicate that these instruments are relevant, at least for the four largest subsamples: materials prices and output prices are each spatially correlated.

Sample	$\hat{\eta}^P$(OLS)	$\hat{\eta}^P$(FE)	$\hat{\eta}^P$(IV)	$\hat{\chi}$	$\hat{\eta}^D$	$\hat{\varepsilon}^Q$(OLS)	$\hat{\varepsilon}^Q$(FE)	$\hat{\varepsilon}^Q$(IV)	N(FE)	N(IV)
Boxes, Yr.≤'87	0.637	0.774	0.347	0.021	2.840	0.683	0.817	0.398	1500	1820
	(0.020)	(0.021)	(0.203)	[0.019,0.023]	(0.118)	[0.623,0.743]		[-0.531,0.769]		
Boxes, Yr.≥'92	0.644	0.776		0.022	1.825	0.670	0.799		549	646
	(0.029)	(0.038)		[0.018,0.025]	(0.148)	[0.599,0.742]				
Coffee	0.682	0.776		0.033	5.744	0.848	0.938		248	300
	(0.046)	(0.058)		[0.026,0.040]	(0.690)	[0.717,0.968]				
Concrete	0.371	0.491	-0.031	0.048	2.577	0.478	0.592	0.096	2047	3708
	(0.019)	(0.031)	(0.061)	[0.045,0.052]	(0.232)	[0.432,0.524]		[-0.027,0.212]		
Flour	0.312	0.652		0.019	1.151	0.328	0.661		452	503
	(0.035)	(0.038)		[0.016,0.021]	(0.351)	[0.224,0.434]				
Gasoline	0.482	0.403	0.496	0.020	3.466	0.540	0.464	0.555	635	692
	(0.054)	(0.065)	(0.184)	[0.016,0.023]	(1.308)	[0.365,0.685]		[0.077,1.005]		
Milk, Bulk	0.810	0.656		0.040	2.380	0.872	0.725		65	127
	(0.085)	(0.171)		[0.028,0.048]	(0.553)	[0.687,1.063]				
Milk, Packaged	0.605	0.731	0.420	0.035	2.555	0.674	0.797	0.496	1534	2099
	(0.020)	(0.025)	(0.161)	[0.032,0.039]	(0.145)	[0.629,0.728]		[0.078,0.874]		
Sugar	1.034	0.922		0.065	2.221	1.114	1.007		155	177
	(0.111)	(0.115)		[0.040,0.085]	(0.826)	[0.765,1.584]				
Yarn	0.551	0.629		0.029	1.525	0.580	0.655		332	431
	(0.037)	(0.050)		[0.025,0.032]	(0.187)	[0.497,0.657]				
Pooled	0.538	0.680	0.078	0.035	2.543	0.608	0.746	0.164	7517	10,503
	(0.010)	(0.012)	(0.053)	[0.033,0.036]	(0.080)	[0.582,0.636]		[0.048,0.279]		

Table 9: Components of the industry-level elasticity of substitution.
Notes: The first three columns present $\hat{\eta}^P$, as estimated using Equation 22. The values given in the fourth, sixth, seventh, and eighth, columns are computed as in Equation 23, while $\hat{\eta}^D$ is estimated using Equation 24. Robust standard errors are included in the first, second, and fourth columns, while bootstrapped confidence intervals are provided in the third, sixth, and eighth columns. "OLS," "FE," and "IV" refer, respectively, to the values corresponding to the ordinary least squares, fixed effects, and instrumental variables specifications for the estimate of η^P.

B.4 The industry-level elasticity of substitution

The previous subsection provided an estimate for the ease with which individual plants substitute between material inputs and other inputs. This is related to, but distinct from, how easily an industry substitutes between material inputs and other inputs.

Changes in the scale, across plants, potentially makes the industry-level elasticity of substitution larger than the corresponding plant-level elasticity. The difference between the plant-level and industry-level elasticities of substitution depends on a) the heterogeneity of materials shares, within the industry, and b) how much inputs shift across plants, in response to a change in relative factor prices.

Given the assumptions, specified in Section B.2, the industry-level elasticity of substitution has a simple expression:[33]

$$
\begin{aligned}
\varepsilon_Q &= \chi_{tI} \cdot \eta_D + (1 - \chi_{tI}) \cdot \eta_P, \text{ where} \qquad (23) \\
\chi_{tI} &\equiv \underbrace{\frac{1}{S_{tI}\left(1 - S_{tI}\right)}}_{①} \cdot \sum_{i \in I} \underbrace{\left(S_{tI} - \frac{M_{it} P_{it}^{in}}{M_{it} P_{it}^{in} + O_{it} P_{it}^{oth}}\right)^2}_{②} \cdot \underbrace{\frac{M_{it} P_{it}^{mat} + O_{it} P_{it}^{oth}}{\sum_{j \in I} M_{jt} P_{jt}^{mat} + O_{jt} P_{jt}^{oth}}}_{③}, \text{ and} \\
S_{tI} &\equiv \sum_{i \in I} \frac{M_{it} P_{it}^{mat}}{M_{it} P_{it}^{mat} + O_{it} P_{it}^{oth}}
\end{aligned}
$$

In words, the industry-level elasticity of substitution is a convex combination of the plant-level elasticity of substitution and the plant-level elasticity of demand. The demand elasticity parameterizes how sensitive the scale of the plant is to changes in its marginal cost of production. Consider, for example, an increase in the price of the material input. The marginal cost of production will increase more for plants with relatively large materials cost shares. As a result, low-materials-share plants will produce relatively more of the total industry output following the increase of the materials price. The elasticity of demand determines how much less the high-materials-share plants will produce, following the increase in the materials price.

The scope for this across-plant factor substitution depends on the dispersion of materials intensities. According to Equation 23, the appropriate measure of the dispersion of materials intensity is a weighted, normalized variance of the materials cost shares. The fraction of total industry expenditures incurred by plant i (given in term ③) is the appropriate weight for summing over the within-industry deviation in materials cost shares (given in term ②). The normalization, given in term ①, ensures the χ_{tI} lies within the unit interval.

What remains, then, is to provide estimates for the normalized variance of materials

[33] A proof is given in Oberfield and Raval (2015). See Appendix A of that paper.

shares, χ, and the elasticity of demand, η_D, for the ten industries in my sample.

The normalized variance of materials shares, χ, ranges from 0.019 (for flour) to 0.065 (for sugar).[34] Given these low values, the industry elasticity of substitution will closely track the micro elasticity of substitution. In other words, the estimate of ε_Q will be, for the most part, insensitive to the way in which η_D is estimated.

I estimate η_D via the regression defined by the following equation:

$$q_{it} = \phi_t + \phi_1 \cdot \log INCOME_{\Upsilon t} + \eta_D \cdot p_{it} + \theta_{it} \tag{24}$$

This specification, and the variable definitions, follow Foster, Haltiwanger, and Syverson (2008). In Equation 24, $INCOME_{\Upsilon t}$ is the aggregate income in establishment i's market, Υ, at time t. This variable is included to account for any differences in establishment scale that may exist between areas of high and low density of economic activity.

A positive relationship between the demand shifter (θ_{it}) and output price (p_{it}) potentially induces a downward bias to the OLS estimates of η_D. Like Foster, Haltiwanger, and Syverson (2008), I instrument p_{it} with the marginal cost of plant i in year t. This instrumental variable is certainly relevant: plants with lower marginal costs have significantly lower output prices. Validity of the instrument rests, then, on the orthogonality of marginal costs and θ_{it}. Foster, Haltiwanger, and Syverson (2008) discuss two potential threats to the validity of the instrument (measurement error in plants' marginal costs, and a selection bias that induces a negative relationship between demand shocks and marginal costs), propose robustness checks to assess the salience of these two threats, and find that their results are similar across the different robustness checks.

The results of these regressions are presented in the fourth column of Table 9.[35] In each of the ten industries, the estimate for elasticity of demand is greater than 1, reassuringly indicating that plants are pricing on the elastic portion of their demand curve.

Combining the estimates of η_P, η_D, and χ yields the object of interest: the industry-level elasticity of substitution, ε_Q. Since there are three sets of estimates of η_P, there are also three sets of estimates of ε_Q. For the estimates corresponding to the fixed effects regression, ε_Q is 0.75 for the pooled sample.[36] Except for sugar and coffee (two of the smallest industries,

[34] To give the reader some idea, the (unnormalized) standard deviations of materials shares range from 4.3 percent to 11.4 percent across the ten industries, again lowest for bulk milk and highest for raw cane sugar.

[35] The results reported here are slightly different from those in Foster, Haltiwanger, and Syverson (2008): I restrict my sample to those plants for which I can observe materials prices, while Foster, Haltiwanger, and Syverson make no such restriction. Their estimate of η_D is lower for petroleum ($\hat{\eta}_D = 1.42$) and higher for ready-mix concrete ($\hat{\eta}_D = 5.93$). Again, because the normalized variances of materials shares are so small, these differences have will have only a moderate impact on the estimates of ε_Q.

[36] One dissimilarity between the analysis of the current section and that of Sections 2 to 4 concerns the industry definitions that I have used: to credibly compare the material purchases and material prices, I

representing only 5 percent of the sample), the industry-level elasticities of substitution range between 0.46 (for gasoline) and 0.82 (for corrugated boxes). For seven of the ten industries in the sample (with the exceptions being the smallest three subsamples), the data would reject a null hypothesis of $\varepsilon_Q = 1$.

The estimates of ε_Q that correspond to the instrumental-variables-based estimate of η_P are smaller, though again much less precisely estimated. The point estimate for ε_Q is 0.1 for the ready-mix concrete subsample, and is somewhat higher (between 0.40 and 0.55) for the other three industries.

In summation, micro data on plants' materials usage patterns indicate that material inputs are gross complements to other factors of production. For most specifications (all except for the IV specification for the ready-mix concrete subsample, or the fixed effects specification for the smaller industries), the data indicate that ε_Q ranges between 0.4 and 0.8.

C Details of the data from outside the U.S.

The data from other countries come from two sources. The flows of intermediate inputs, flows of goods output into final consumption expenditures, and industry-level prices are collected in the World Input Output Tables (WIOT). The data on industries' output are compiled in the European Union KLEMS Growth and Productivity Accounts (EUKLEMS).[37] The EUKLEMS data are reviewed, in detail, in Timmer et al. (2007) and O'Mahony and Timmer (2009). Flows of investment goods across industries are not available for other countries. For this set of variables, I imputed values using data from the U.S.

Of the thirty countries that are included in the EUKLEMS dataset, I restrict my analysis to six: Denmark, France, Italy, Japan, the Netherlands, and Spain. Many of the countries that I discarded are Eastern Bloc countries—such as Latvia, Lithuania, and Poland—for which pre-1990 data are unavailable. There are other countries, such as England, for which—for at least half of the sample period—intermediate input purchases and gross output are imputed from value added data. Data from all countries span 1970 to 2007, with the excep-

define products narrowly in this section. At the same time, limitations of the dataset necessitate a rather coarse industry definition in Sections 2 to 4. Going from a narrow to coarse industry classification should not systematically alter the estimates of η_P or η_D, but will cause an increase in the estimate for the within-industry variation in materials cost shares, χ. For this reason, a coarser industry classification would, in turn, lead to a larger estimate of ε_Q. As it turns out, the overall estimate of ε_Q is not particularly sensitive to the value of χ: Doubling the value of χ increases the OLS-based estimate of ε_Q from 0.61 to 0.67, and increases the fixed-effects-based estimate from 0.75 to 0.80.

[37] The data can be downloaded at http://www.euklems.net/ . In this section I use the ISIC Rev. 3 edition of the data.

#	Name	Denmark	France	Italy	Japan	Netherlands	Spain	U.S. Ind.
1	Agriculture	0.027	0.029	0.022	0.010	0.036	0.039	1
2	Mining	0.007	0.002	0.001	0.000	0.010	0.001	2,3
3	Food and Tobacco	0.103	0.061	0.065	0.069	0.098	0.085	5
4	Textiles and Leather	0.018	0.025	0.069	0.019	0.020	0.039	6, 7
5	Wood Products	0.007	0.002	0.003	0.002	0.004	0.002	8, 9
6	Paper and Publishing	0.017	0.014	0.015	0.002	0.022	0.015	10, 11
7	Petroleum Refining	0.012	0.011	0.013	0.010	0.024	0.016	13
8	Chemicals	0.040	0.047	0.037	0.016	0.073	0.038	12
9	Rubber and Plastics	0.013	0.009	0.012	0.005	0.012	0.008	14
10	Stone, Clay, and Glass	0.007	0.005	0.010	0.002	0.006	0.008	15
11	Metal products	0.021	0.018	0.024	0.015	0.029	0.020	16, 17
12	Non-Electrical Machinery	0.047	0.024	0.048	0.028	0.026	0.018	18
13	Electrical Machinery	0.038	0.039	0.027	0.042	0.040	0.024	19, 22
14	Transportation Equipment	0.026	0.065	0.040	0.040	0.034	0.074	20, 21
15	Misc. Manufacturing	0.024	0.013	0.024	0.007	0.018	0.020	23
16	Utilities	0.021	0.037	0.016	0.017	0.017	0.015	26
17	Construction	0.010	0.006	0.008	0.000	0.006	0.006	4
18	Wholesale and Retail	0.073	0.078	0.109	0.137	0.070	0.075	27
19	Hotels and Restaurants	0.020	0.031	0.051	0.048	0.021	0.106	28
20	Transport and Warehousing	0.060	0.038	0.048	0.043	0.062	0.037	24
21	Communications	0.010	0.010	0.012	0.014	0.012	0.012	25
22	Finance and Insurance	0.021	0.032	0.026	0.032	0.032	0.015	28
23	Real Estate	0.086	0.100	0.072	0.127	0.060	0.069	28
24	Business Services	0.015	0.030	0.018	0.009	0.052	0.017	29
25	Government	0.071	0.096	0.079	0.105	0.079	0.076	30
26	Education	0.058	0.056	0.053	0.060	0.035	0.052	29
27	Health and Social Work	0.111	0.087	0.065	0.082	0.070	0.066	29
28	Other Personal Services	0.038	0.035	0.032	0.058	0.030	0.048	29

Table 10: Industry definitions and consumption shares in the EUKLEMS dataset.
Notes: The final column shows the correspondence between the EUKLEMS industry definitions and the industry definitions for the U.S. data.

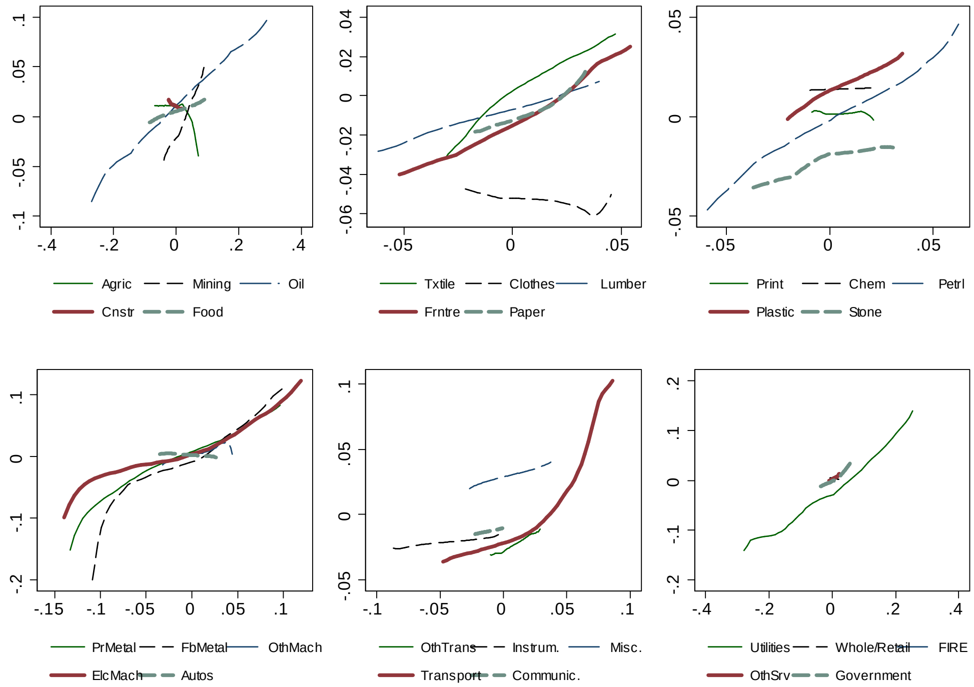

Figure 5: Relationship between changes in intermediate input purchases and intermediate input prices.

Notes: For each downstream industry, J, I take the most important (highest average intermediate input expenditure share) supplier industry, I. The x-axis of each panel gives $\Delta\log\left(\frac{P_{tI}}{P_{tJ}^{in}}\right)$. The y-axis gives, for each industry, changes in the fraction of industry J's intermediate input expenditures that go to industry I. I compute and plot a local polynomial curve of this relationship, for each industry.

tion of Japan, whose sample begins in 1973.

The industry definitions in the EUKLEMS database differ from those in the U.S. dataset. Service industries are more finely defined. For example, F.I.R.E. is now broken out between finance and insurance on the one hand and real estate on the other. Mining and manufacturing industries are more coarse. Table 10 describes the EUKLEMS industry classification, in addition to the consumption shares of each of the 28 industries. The main takeaway from this table is that the six countries are broadly similar in their industry compositions.

D Sensitivity analysis related to Section 3

D.1 Additional Plots

Figure 5 depicts the smoothed relationship between $\Delta\log\left(\frac{P_{tI}M_{t,I\to J}}{P_{tJ}^{in}M_{tJ}}\right)$ and $\Delta\log\left(\frac{P_{tI}}{P_{tJ}^{in}}\right)$, for each industry J and J's most important supplier industry. The takeaway from this figure

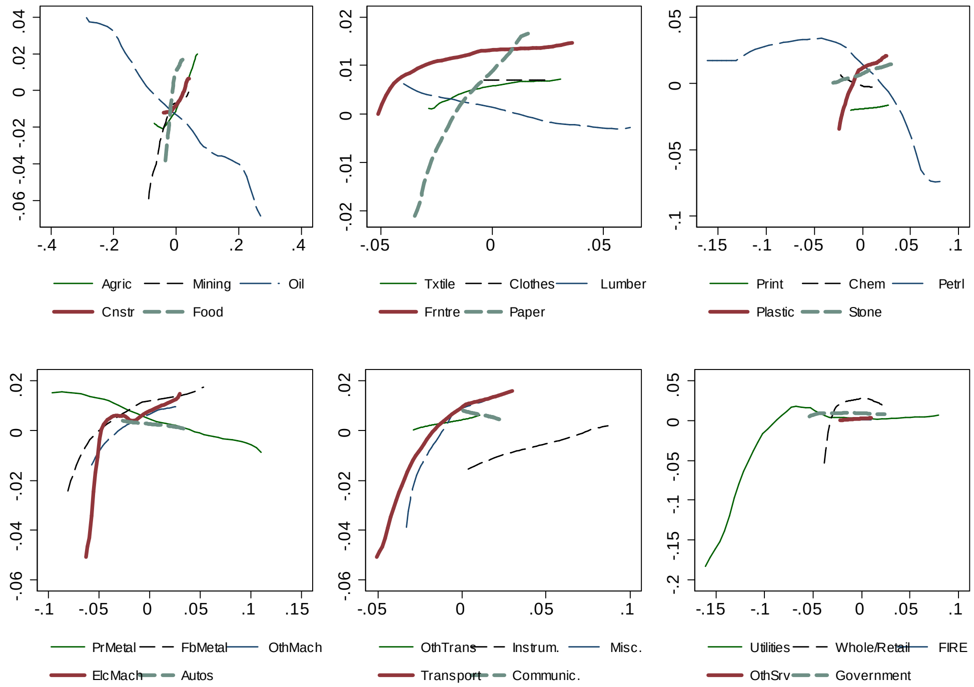

Figure 6: Relationship between changes in purchases of the intermediate input bundle and the relative price of the intermediate input bundle.
Notes: For each industry, J, I plot the relationship between changes in its cost share of intermediate inputs on the y-axis, and changes in the difference between the price of the intermediate input bundle and the marginal cost of production on the x-axis. I compute and plot a local polynomial curve of this relationship for each industry.

is that the relationships depicted in Figure 1 are broadly consistent with of the relationships within all 30 industries.

Figure 6 depicts the smoothed relationship between $\Delta \log\left(\frac{P^{in}_{tJ} M_{tJ}}{P_{tJ} Q_{tJ}}\right)$ and $\Delta \log\left(\frac{P^{in}_{tJ}}{P_{tJ}}\right)$. Here, the relationship between intermediate input cost shares and the price of intermediate inputs is positive for some industries, negative for others.

D.2 Different samples, changing the period length and industry classification

In this section, I re-estimate Equation 13 using different samples. First, in Table 11, I examine whether the estimates of the production elasticities, ε_Q and ε_M, differ according to the industry classification scheme or the period length. In the first four columns, the economy is broken up into nine industries; in the next four columns, a 67-industry classification is applied. The main takeaway from this table is that the estimates of ε_M, as in the original specifications, are close to 0, independent of how industries are defined. For longer period lengths, the estimated elasticity of substitution among intermediate inputs is somewhat higher; the elasticity of substitution between value added and intermediate inputs is somewhat lower. The IV results are unreported for this last robustness check, since the instruments are both weak and lead one to reject the Wu-Hausman test.

Next, in Table 12, I estimate the production elasticities separately for different broad sectors. The Primary sector consists of the first three industries in Table 7. The Manufacturing sector consists of the Construction and all manufacturing industries, the fourth through twenty-third industries according to Table 7. The remaining industries are in the Services sector. Estimates of ε_M are similar across sectors. Estimates of ε_Q, though less precisely estimated, are somewhat larger for the Primary sector and lower for the Services sector.

As a third set of robustness checks, I assess in Table 13 whether the number of upstream industries used in the sample alters the estimates of ε_M and ε_Q. In the benchmark regressions, in Table 1, the sample included the top ten upstream industries for each downstream industry J. There are no clear patterns, regarding the relationship between estimates of ε_Q and ε_M and the broadness of the sample.

D.3 Production elasticities of substitution in other countries

In this subsection, I report on results from other countries. I apply data from the World Input Output Tables, taking data from 1997 to 2011. The industry definitions, similar those used for the U.S. data, are given in Table 10. In Table 14, I report on regressions that relate changes in the inputs' cost shares with changes in the prices of individual inputs and prices of the intermediate input bundles. Unfortunately, for these countries, changes in military expenditures are not a sufficiently powerful source of variation to permit an IV regression. In this table, the slope of the relationship of changes in the intermediate input cost share on $\Delta \log P_{tJ}^{in} - \Delta \log P_{tI}$ is approximately 0.3 for France and between 0.6 and 0.8 for all other countries. In addition, the slope of the relationship of intermediate input purchases on $\Delta \log P_{tJ} - \Delta \log P_{tJ}^{in}$ is 0.10 for Denmark and between 0.4 and 0.8 for all other countries. While the coefficient estimates reported in 14 cannot identify ε_Q or ε_M on their own, they accord with the OLS estimates for the United States.

E Sensitivity analysis related to Section 4

In the first columns of Table 15, I re-estimate the correlations among shocks for different parts of the sample period. For the most part, the correlations among the ω productivity

Second stage regression results										
ε_M	0.17	0.13	0.20	0.22	0.16	0.12	-0.06	-0.03	0.29	0.29
	(0.05)	(0.05)	(0.28)	(0.27)	(0.03)	(0.03)	(0.22)	(0.24)	(0.07)	(0.07)
ε_Q	1.08	1.13	0.55	1.19	1.03	1.13	1.03	1.09	0.95	0.95
	(0.08)	(0.08)	(0.55)	(0.38)	(0.05)	(0.05)	(0.66)	(1.04)	(0.12)	(0.12)
First stage: Dependent variable is $\Delta \log P^{in}_{tJ} - \Delta \log P_{tI}$										
military spending			-0.56	-0.55			-0.44	-0.37		
shock_{tI}			(0.06)	(0.06)			(0.03)	(0.03)		
military spending			0.62	0.69			0.53	0.76		
$\text{shock}_{tJ\text{'s suppliers}}$			(0.09)	(0.12)			(0.04)	(0.06)		
military spending			-0.05	-0.06			0.01	-0.02		
shock_{tJ}			(0.07)	(0.07)			(0.03)	(0.03)		
F-statistic			32.29	14.92			88.57	29.78		
First stage: Dependent variable is $\Delta \log P_{tJ} - \Delta \log P^{in}_{tJ}$										
military spending			-0.11	0.01			-0.07	-0.05		
shock_{tI}			(0.04)	(0.04)			(0.02)	(0.02)		
military spending			-0.18	0.19			-0.09	0.00		
$\text{shock}_{tJ\text{'s suppliers}}$			(0.07)	(0.08)			(0.03)	(0.04)		
military spending			0.30	0.27			0.11	0.09		
shock_{tJ}			(0.05)	(0.05)			(0.02)	(0.02)		
F-statistic			17.83	13.36			21.37	26.92		
Cragg-Donald Statistic			15.34[i]	27.60[i]			21.37[i]	8.35		
Wu-Hausman test p-value			0.59	0.90			0.70	0.87		
Sample	Coarse Industries				Fine Industries				Period Length = 2 Years	
Year Fixed Effects	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes
N	2400	2400	2296	2296	10720	10720	10496	10496	1296	1296

Table 11: Regression results related to Equation 13.

Notes: The overall sample includes pairs of industries J, and, for each industry J, the top ten supplying industries, I. In the row labeled "Cragg-Donald Statistic", an "i" indicates that the test for a weak instrument is rejected at the 10 percent threshold. The "military spending $\text{shock}_{tJ\text{'s suppliers}}$" term refers to the cost-weighted average of the "military spending shock_{tI}" term, averaging over industry J's suppliers.

Second stage regression results									
ε_M	0.02	-0.38	-0.35	-0.27	-0.27	-0.04	0.26	0.39	0.35
	(0.11)	(0.36)	(0.35)	(0.05)	(0.28)	(0.36)	(0.08)	(0.48)	(0.43)
ε_Q	1.41	1.44	1.64	1.35	1.55	0.49	0.02	0.93	0.90
	(0.10)	(0.36)	(0.42)	(0.09)	(1.15)	(0.74)	(0.15)	(0.91)	(0.89)
First stage: Dependent variable is $\Delta \log P_{tJ}^{in} - \Delta \log P_{tI}$									
military spending		-1.39	-1.43		-0.58	-0.48		-0.76	-0.77
shock$_{tI}$		(0.20)	(0.21)		(0.07)	(0.08)		(0.13)	(0.13)
military spending		1.55	-0.06		0.97	1.21		0.92	0.93
shock$_{tJ\text{'s suppliers}}$		(0.49)	(1.41)		(0.11)	(0.13)		(0.22)	(0.26)
military spending		-0.09	0.46		-0.25	-0.25		-0.22	-0.26
shock$_{tJ}$		(0.27)	(0.52)		(0.08)	(0.08)		(0.27)	(0.31)
F-statistic		16.78	4.59		38.36	11.41		13.13	3.20
First stage: Dependent variable is $\Delta \log P_{tJ} - \Delta \log P_{tJ}^{in}$									
military spending		0.01	0.02		-0.16	0.03		0.03	-0.03
shock$_{tI}$		(0.22)	(0.19)		(0.04)	(0.04)		(0.07)	(0.06)
military spending		-0.59	-3.03		0.12	0.55		-0.46	-0.73
shock$_{tJ\text{'s suppliers}}$		(0.52)	(1.31)		(0.06)	(0.07)		(0.12)	(0.12)
military spending		0.94	1.81		-0.04	-0.03		0.27	0.00
shock$_{tJ}$		(0.29)	(0.49)		(0.04)	(0.04)		(0.15)	(0.14)
F-statistic		15.36	13.36		7.99	21.98		12.15	26.54
Cragg-Donald Statistic		15.25[i]	12.79		6.02	11.42		11.39	12.73
Wu-Hausman test p-value		0.49	0.50		0.98	0.37		0.53	0.99
Sector		Primary			Manufacturing			Services	
Year Fixed Effects	No	No	Yes	No	No	Yes	No	No	Yes
N	480	480	480	3200	3200	3200	1120	912	912

Table 12: Regression results related to Equation 13.

Notes: The overall sample includes pairs of industries I-J that, for each industry J, I include J's top ten supplying industries, I. In the row labeled "Cragg-Donald Statistic", an "i" indicates that the test for a weak instrument is rejected at the 10 percent threshold. The "military spending shock$_{tJ\text{'s suppliers}}$" term refers to the cost-weighted average of the "military spending shock$_{tI}$" term, averaging over industry J's suppliers.

Second stage regression results						
ε_M	-0.14	-0.09	-0.05	-0.06	-0.10	-0.08
	(0.23)	(0.25)	(0.19)	(0.19)	(0.16)	(0.16)
ε_Q	0.82	0.42	0.73	0.88	0.98	0.88
	(0.56)	(0.46)	(0.45)	(0.36)	(0.41)	(0.33)
First stage: Dependent variable is $\Delta \log P^{in}_{tJ} - \Delta \log P_{tI}$						
military spending shock$_{tI}$	-0.89	-0.86	-0.86	-0.80	-0.81	-0.74
	(0.09)	(0.09)	(0.07)	(0.07)	(0.05)	(0.06)
military spending shock$_{tJ\text{'s suppliers}}$	0.82	0.95	1.02	1.22	1.15	1.35
	(0.13)	(0.17)	(0.10)	(0.13)	(0.09)	0.12)
military spending shock$_{tJ}$	0.11	0.11	-0.12	-0.13	-0.20	-0.22
	(0.09)	(0.09)	(0.07)	(0.07)	(0.06)	0.06)
F-statistic	41.09	10.84	63.80	15.01	95.92	20.11
First stage: Dependent variable is $\Delta \log P_{tJ} - \Delta \log P^{in}_{tJ}$						
military spending shock$_{tI}$	-0.17	-0.10	-0.14	-0.03	-0.12	0.02
	(0.07)	(0.07)	(0.05)	(0.05)	(0.03)	0.03)
military spending shock$_{tJ\text{'s suppliers}}$	-0.28	0.14	-0.29	0.11	-0.30	0.10
	(0.10)	(0.13)	(0.07)	(0.09)	(0.05)	0.07)
military spending shock$_{tJ}$	0.40	0.36	0.38	0.35	0.38	0.35
	(0.07)	(0.07)	(0.05)	(0.05)	(0.04)	0.04)
F-statistic	12.38	5.11	22.81	9.80	43.52	18.17
Cragg-Donald Statistic	9.43	13.55[i]	21.57[i]	32.33[i]	42.18[i]	62.21[i]
Wu-Hausman test p-value	0.85	0.32	0.56	0.56	0.37	0.16
Year Fixed Effects	No	Yes	No	Yes	No	Yes
Upstream Industries per downstream industry×year	4	4	8	8	15	15
N	1856	1856	3680	3680	6832	6832

Table 13: Regression results related to Equation 13.

Notes: The overall sample includes pairs of industries J, and, for each industry J, the top two supplying industries, I in the first four columns, and the top four supplying industries in the final four columns. In the row labeled "Cragg-Donald Statistic", an "$^{\text{i}}$" indicates that the test for a weak instrument is rejected at the 10 percent threshold. The "military spending shock$_{tJ\text{'s suppliers}}$" term refers to the cost-weighted average of the "military spending shock$_{tI}$" term, averaging over industry J's suppliers.

ε_M	0.28	0.36	0.70	0.19	0.42	0.30
	(0.05)	(0.05)	(0.05)	(0.05)	(0.04)	(0.04)
ε_Q	0.11	0.56	0.71	0.81	0.79	0.47
	(0.06)	(0.05)	(0.04)	(0.05)	(0.04)	(0.05)
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
N	3920	3920	3920	3920	3920	3920
Country	DNK	ESP	FRA	ITA	JPN	NLD

Table 14: Regression results related to Equation 13.

Notes: This table contains OLS specifications, using ten input-supplying industries per downstream industry.

	Bench -mark	1960- 1983	1984- 2012	1960- 2007	Period Length is 2 years	Durable Goods
R^2 (sectoral shocks)						
$(\varepsilon_M, \varepsilon_D, \varepsilon_Q) = (1, 1, 1)$	0.21^{kf}			0.32^{kf}		0.21
$(\varepsilon_M, \varepsilon_D, \varepsilon_Q) = \left(1, 1, \frac{4}{5}\right)$	0.19^{kf}			0.29^{kf}		0.19
$(\varepsilon_M, \varepsilon_D, \varepsilon_Q) = \left(\frac{1}{10}, 1, \frac{4}{5}\right)$	0.81			0.81		0.97^{kf}
$(\varepsilon_M, \varepsilon_D, \varepsilon_Q) = \left(\frac{1}{10}, \frac{2}{3}, 1\right)$	0.99^{kf}			1.00^{kf}		0.91^{kf}
$(\varepsilon_M, \varepsilon_D, \varepsilon_Q) = \left(\frac{1}{10}, 1, 1\right)$	0.83			0.94		0.98^{kf}
$(\varepsilon_M, \varepsilon_D, \varepsilon_Q) = \left(\frac{1}{10}, \frac{4}{3}, 1\right)$	0.59			0.71		0.92^{kf}
$\bar{\rho}\,(\omega)$						
$(\varepsilon_M, \varepsilon_D, \varepsilon_Q) = (1, 1, 1)$	0.19^{kf}	0.19^{kf}	0.19^{kf}	0.17^{kf}	0.18	0.20
$(\varepsilon_M, \varepsilon_D, \varepsilon_Q) = \left(1, 1, \frac{4}{5}\right)$	0.21^{kf}	0.21^{kf}	0.20^{kf}	0.18^{kf}	0.19	0.22
$(\varepsilon_M, \varepsilon_D, \varepsilon_Q) = \left(\frac{1}{10}, 1, \frac{4}{5}\right)$	0.06	0.06	0.09	0.06	0.05^{kf}	0.06^{kf}
$(\varepsilon_M, \varepsilon_D, \varepsilon_Q) = \left(\frac{1}{10}, \frac{2}{3}, 1\right)$	0.04^{kf}	0.03^{kf}	0.04^{kf}	0.03^{kf}	0.04^{kf}	0.05^{kf}
$(\varepsilon_M, \varepsilon_D, \varepsilon_Q) = \left(\frac{1}{10}, 1, 1\right)$	0.06	0.05	0.07	0.05	0.06^{kf}	0.06^{kf}
$(\varepsilon_M, \varepsilon_D, \varepsilon_Q) = \left(\frac{1}{10}, \frac{4}{3}, 1\right)$	0.08	0.07	0.07	0.07	0.08	0.06^{kf}

Table 15: Robustness checks: R^2(sectoral shocks) and $\bar{\rho}(\omega)$ for different values of ε_D, ε_M, and ε_Q.

Notes: I could not compute R^2(sectoral shocks) in the second, third, and fifth columns, as there are fewer time periods than there are industries in these samples. A "kf" indicates the use of the Kalman filter, as opposed to direct applications of Equation 11, to infer the ω productivity shocks.

shocks are similar in the first half and the second half of the sample. (Unfortunately, since there are fewer time periods in either of the two halves of the sample than there are industries, I cannot compute the first factor of the industries' productivity shocks to assess the contribution of common productivity shocks to aggregate volatility.) In the fourth column, I exclude the Great-Recession period from the sample. Here, the assessed role of industry-specific shocks is somewhat larger. The fifth column applies biennial data. The final column incorporates good durability, in which I allow for certain industries' outputs to depreciate over a number of periods. In this column, I set $\delta_{C_J} = 1$ for all nondurable industries and $\delta_{C_J} = 0.4$ for durable industries. In this column, sectoral shocks now constitute a larger fraction of aggregate volatility when $\varepsilon_M = \frac{1}{10}$.[38]

A final set of robustness check considers the sensitivity of the main results to the de-trending procedure.[39] In the benchmark calculations, I had linearly de-trended each

[38] These depreciation rates are considerably larger than have been estimated elsewhere by, for example, Hulten and Wykoff (1981). Unfortunately, applying lower depreciation rates would lead to exceedingly large eigenvalues of $(\Pi_3)^{-1}\Pi_2$.

[39] In estimations of dynamic general equilibrium models, the choice of the de-trending procedure is potentially important; see Canova (2014). An alternative—intuitively appealing but unfortunately infeasible—way to deal with trends would be to include both transitory and permanent shocks in the model. This would obviate the need to de-trend the data before estimation; the parameters governing the permanent and transitory

De-trending Method	Benchmark	None	Hodrick-Prescott	Linear, Break in 1983	Linear, Censor Outliers
R^2 (sectoral shocks)					
$(\varepsilon_M, \varepsilon_D, \varepsilon_Q) = (1, 1, 1)$	0.21^{kf}	0.22^{kf}	0.22^{kf}	0.20^{kf}	0.22^{kf}
$(\varepsilon_M, \varepsilon_D, \varepsilon_Q) = \left(1, 1, \frac{4}{5}\right)$	0.19^{kf}	0.20^{kf}	0.19^{kf}	0.18^{kf}	0.20^{kf}
$(\varepsilon_M, \varepsilon_D, \varepsilon_Q) = \left(\frac{1}{10}, 1, \frac{4}{5}\right)$	0.81	0.80	0.70	0.79	0.85
$(\varepsilon_M, \varepsilon_D, \varepsilon_Q) = \left(\frac{1}{10}, \frac{2}{3}, 1\right)$	0.99^{kf}	0.99^{kf}	0.99^{kf}	0.99^{kf}	1.00^{kf}
$(\varepsilon_M, \varepsilon_D, \varepsilon_Q) = \left(\frac{1}{10}, 1, 1\right)$	0.83	0.83	0.76	0.82	0.77
$(\varepsilon_M, \varepsilon_D, \varepsilon_Q) = \left(\frac{1}{10}, \frac{4}{3}, 1\right)$	0.59	0.58	0.53	0.57	0.60
$\bar{\rho}(\omega)$					
$(\varepsilon_M, \varepsilon_D, \varepsilon_Q) = (1, 1, 1)$	0.19^{kf}	0.19^{kf}	0.20^{kf}	0.20^{kf}	0.20^{kf}
$(\varepsilon_M, \varepsilon_D, \varepsilon_Q) = \left(1, 1, \frac{4}{5}\right)$	0.21^{kf}	0.21^{kf}	0.21^{kf}	0.21^{kf}	0.21^{kf}
$(\varepsilon_M, \varepsilon_D, \varepsilon_Q) = \left(\frac{1}{10}, 1, \frac{4}{5}\right)$	0.06	0.05	0.05	0.06	0.06
$(\varepsilon_M, \varepsilon_D, \varepsilon_Q) = \left(\frac{1}{10}, \frac{2}{3}, 1\right)$	0.04^{kf}	0.04^{kf}	0.04^{kf}	0.04^{kf}	0.03^{kf}
$(\varepsilon_M, \varepsilon_D, \varepsilon_Q) = \left(\frac{1}{10}, 1, 1\right)$	0.06	0.05	0.06	0.06	0.05
$(\varepsilon_M, \varepsilon_D, \varepsilon_Q) = \left(\frac{1}{10}, \frac{4}{3}, 1\right)$	0.08	0.08	0.07	0.08	0.07

Table 16: Robustness checks: R^2(sectoral shocks) and $\bar{\rho}(\omega)$ for different values of ε_D, ε_M, and ε_Q.

Notes: A "$^{\text{kf}}$" indicates the use of the Kalman filter, as opposed to direct applications of Equation 11, to infer the ω productivity shocks.

industry-level observable before performing the filtering exercise. In Table 16, I consider three alternate de-trending procedures: not de-trending the data, a Hodrick-Prescott filter, and a linear trend with a break in the trend at 1983. These de-trending procedures have almost no quantitative impact on the relative contribution of sectoral vs. common shocks to aggregate volatility. Finally, censoring outlier observations (those industry-year output growth rates in the top or bottom centile) does not alter the estimated importance of sectoral shocks.

F Solution of the model filter

This section spells out the solution of the model. First, I write out the constrained maximization problem of a social planner. I take first-order conditions, write out the conditions that characterize the steady state, log-linearize around the steady state, solve for the policy functions, and then for the model filter. We allow not only for factor neutral productivity shocks (as used throughout the paper), but also labor-augmenting productivity shocks, as

shock processes would be jointly estimated in a single stage. I do not pursue this approach, mainly because of the difficulty of scaling the model by the permanent shocks. Doing so requires a clean characterization of the changes in the industry-level observable variables as functions of the permanent shocks, something that exists only for a few special cases of the model (such special cases can be found in, for example, Ngai and Pissarides 2007 and Acemoglu and Guerreri 2008).

in the specification in Table 4.

F.1 First order conditions and steady-state shares

Since this economy satisfies the welfare theorems, it will suffice to solve the social planner's problem. Begin with the Lagrangian:

$$\begin{aligned}\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ \log \left[\left[\sum_{J=1}^{N} (\xi_J)^{\frac{1}{\varepsilon_D}} \left(\delta_{C_J} \cdot C_{tJ}\right)^{\frac{\varepsilon_D - 1}{\varepsilon_D}} \right]^{\frac{\varepsilon_D}{\varepsilon_D - 1}} \right] \\ - \frac{\varepsilon_{LS}}{\varepsilon_{LS}+1} \left(\sum_{J=1}^{N} L_{tJ} \right)^{\frac{\varepsilon_{LS}+1}{\varepsilon_{LS}}} + \sum_{J=1}^{N} P_{tJ}^{inv} \left[X_{tJ} + (1-\delta_K) K_{tJ} - K_{t+1,J} \right] \\ + \sum_{J=1}^{N} P_{tJ} \left[Q_{tJ} + \left(1-\delta_{C_J}\right) C_{tJ} - C_{t+1,J} - \sum_{I=1}^{N} \left[M_{t,J\to I} + X_{t,J\to I} \right] \right] \Bigg\} . \end{aligned} \tag{25}$$

Here, P_{tJ}^{inv} is the Lagrange multiplier on a unit of capital, and P_{tJ} is the Lagrange multiplier on the good-J market-clearing condition.

This Lagrangian incorporates durability for some consumption goods, something that was ignored in the body of the paper. The Lagrangian reflects a representative consumer who has preferences given by the following utility function:

$$\mathcal{U} = \sum_{t=0}^{\infty} \beta^t \log \left[\left[\sum_{J=1}^{N} (\xi_J)^{\frac{1}{\varepsilon_D}} \left(\delta_{C_J} \cdot C_{tJ}\right)^{\frac{\varepsilon_D - 1}{\varepsilon_D}} \right]^{\frac{\varepsilon_D}{\varepsilon_D - 1}} \right] - \frac{\varepsilon_{LS}}{\varepsilon_{LS}+1} \left(\sum_{J=1}^{N} L_{tJ} \right)^{\frac{\varepsilon_{LS}+1}{\varepsilon_{LS}}}$$

The demand parameters, ξ_J, again reflect time-invariant differences in the importance of industries' goods in the consumer's preferences. Now, C_{tJ} equals the stock of durable goods when J is a durable-good-producing industry and equals the expenditures on good/service J otherwise. For durable goods, J, the evolution of the stock of each consumption good C_{tJ} is given by

$$C_{t+1,J} = C_{tJ}\left(1-\delta_{C_J}\right) + \tilde{C}_{tJ} \ ,$$

where $\tilde{C}_{tJ}$ equals the consumer's new purchases on good J at time t and δ_{C_J} parameterizes the depreciation rate of good J.

I re-state the expression for Q_{tJ}:

$$Q_{tJ} = A_{tJ} \cdot \left[(1-\mu_J)^{\frac{1}{\varepsilon_Q}} \left(\left(\frac{K_{tJ}}{\alpha_J} \right)^{\alpha_J} \left(\frac{L_{tJ} \cdot B_{tJ}}{1-\alpha_J} \right)^{1-\alpha_J} \right)^{\frac{\varepsilon_Q - 1}{\varepsilon_Q}} + (\mu_J)^{\frac{1}{\varepsilon_Q}} \left(M_{tJ}\right)^{\frac{\varepsilon_Q - 1}{\varepsilon_Q}} \right]^{\frac{\varepsilon_Q}{\varepsilon_Q - 1}} . \tag{26}$$

The first-order conditions for the planner are:

$$
\begin{aligned}
K_{t+1,J} &= X_{tJ} + (1-\delta_K)\cdot K_{tJ} \\
[C_{tJ}] &: P_{t-1,J} \; - \beta P_{tJ}\left(1-\delta_{C_J}\right) = \beta\left(\xi_J\right)^{\frac{1}{\varepsilon_D}}\left(\delta_{C_J}\right)^{\frac{\varepsilon_D-1}{\varepsilon_D}} \times && (27)\\
& \left(C_{tJ}\right)^{-\frac{1}{\varepsilon_D}}\left(\sum_{I=1}^{N}\left(\xi_I\right)^{\frac{1}{\varepsilon_D}}\left(\delta_{C_I}\cdot C_{tI}\right)^{\frac{\varepsilon_D-1}{\varepsilon_D}}\right)^{-1} . && (28)\\
[M_{t,I\to J}] &: \frac{P_{tI}}{P_{tJ}} = \left(A_{tJ}\right)^{\frac{\varepsilon_Q-1}{\varepsilon_Q}}\left(\frac{Q_{tJ}\cdot\mu_J}{M_{tJ}}\right)^{\frac{1}{\varepsilon_Q}}\left(\frac{M_{tJ}\cdot\Gamma^M_{IJ}}{M_{t,I\to J}}\right)^{\frac{1}{\varepsilon_M}} . && (29)\\
[X_{t,I\to J}] &: P_{tI} = P^{inv}_{tJ}\left(\frac{X_{tJ}\cdot\Gamma^X_{IJ}}{X_{t,I\to J}}\right)^{\frac{1}{\varepsilon_X}} . && (30)\\
[L_{tJ}] &: \left(\sum_{J'=1}^{N} L_{tJ'}\right)^{\frac{1}{\varepsilon_{LS}}} = P_{tJ}\cdot\left(A_{tJ}\right)^{\frac{\varepsilon_Q-1}{\varepsilon_Q}} B_{tJ}\left(Q_{tJ}\left(1-\mu_J\right)\right)^{\frac{1}{\varepsilon_Q}} \times && (31)\\
& \left(\frac{K_{tJ}}{\alpha_J}\right)^{\alpha_J\frac{\varepsilon_Q-1}{\varepsilon_Q}}\left(\frac{L_{tJ}\cdot B_{tJ}}{1-\alpha_J}\right)^{\frac{\alpha_J-1-\alpha_J\varepsilon_Q}{\varepsilon_Q}} . \\
[K_{t+1,J}] &: P^{inv}_{tJ} = \beta\cdot\mathbb{E}_t\Bigg[P_{t+1,J}\left(Q_{t+1,J}\left(1-\mu_J\right)\right)^{\frac{1}{\varepsilon_Q}}\left(A_{t+1,J}\right)^{\frac{\varepsilon_Q-1}{\varepsilon_Q}} && (32)\\
& \times\left(\frac{K_{t+1,J}}{\alpha_J}\right)^{-1+\alpha_J\cdot\frac{\varepsilon_Q-1}{\varepsilon_Q}}\left(\frac{L_{t+1,J}\cdot B_{t+1,J}}{1-\alpha_J}\right)^{(1-\alpha_J)\cdot\frac{\varepsilon_Q-1}{\varepsilon_Q}}\Bigg] \\
& +\beta(1-\delta_K)\mathbb{E}_t\left[P^{inv}_{t+1,J}\right] .
\end{aligned}
$$

Towards the goal of solving for the steady-state, drop time subscripts and re-arrange. Also, employ the normalization that steady-state labor is the numeraire good (so that

$\sum_{J'=1}^{N} L_{J'}^{\frac{1}{\varepsilon_{LS}}} = 1)$:

$$\begin{aligned}
\delta_K K_J &= X_J \\
\frac{1-\beta\left(1-\delta_{C_J}\right)}{\beta} P_J &= \left(\xi_J\right)^{\frac{1}{\varepsilon_D}} \cdot \left(\delta_{C_J}\right)^{\frac{\varepsilon_D-1}{\varepsilon_D}} \left(C_J\right)^{-\frac{1}{\varepsilon_D}} \left(\sum_{I=1}^{N} \left(\xi_I\right)^{\frac{1}{\varepsilon_D}} \left(\delta_{C_I} \cdot C_I\right)^{\frac{\varepsilon_D-1}{\varepsilon_D}}\right)^{-1} \\
\frac{P_I}{P_J} &= \left(\frac{Q_J \cdot \mu_J}{M_J}\right)^{\frac{1}{\varepsilon_Q}} \left(\frac{M_J \cdot \Gamma_{IJ}^M}{M_{I\to J}}\right)^{\frac{1}{\varepsilon_M}} \\
P_J^{inv} &= P_I \left(\frac{X_J \cdot \Gamma_{IJ}^X}{X_{I\to J}}\right)^{-\frac{1}{\varepsilon_X}} \\
1 &= P_J \left(Q_J \left(1-\mu_J\right)\right)^{\frac{1}{\varepsilon_Q}} \left(\frac{K_J}{\alpha_J}\right)^{\alpha_J \cdot \frac{\varepsilon_Q-1}{\varepsilon_Q}} \left(\frac{L_J}{1-\alpha_J}\right)^{(1-\alpha_J)\cdot\frac{\varepsilon_Q-1}{\varepsilon_Q}-1} \\
Q_J &= \left[\left(1-\mu_J\right)^{\frac{1}{\varepsilon_Q}} \left(\left(\frac{K_J}{\alpha_J}\right)^{\alpha_J} \left(\frac{L_J}{1-\alpha_J}\right)^{1-\alpha_J}\right)^{\frac{\varepsilon_Q-1}{\varepsilon_Q}} + \left(\mu_J\right)^{\frac{1}{\varepsilon_Q}} \left(M_J\right)^{\frac{\varepsilon_Q-1}{\varepsilon_Q}}\right]^{\frac{\varepsilon_Q}{\varepsilon_Q-1}}
\end{aligned} \tag{33}$$

First, I will solve for the prices of each industry's good, in the steady state, P_J. This will follow from each industry's cost-minimization condition.

Take the cost-minimization condition for capital, which equates the rental price of a unit of capital to the marginal revenue product of capital:

$$\frac{1-\beta(1-\delta_K)}{\beta} \left[\sum \Gamma_{IJ}^X \left(P_I\right)^{1-\varepsilon_X}\right]^{1/(1-\varepsilon_X)} = P_J \left(Q_J \left(1-\mu_J\right)\right)^{\frac{1}{\varepsilon_Q}} \left(\frac{K_J}{\alpha_J}\right)^{\alpha_J \cdot \frac{\varepsilon_Q-1}{\varepsilon_Q}-1} \left(\frac{L_J}{1-\alpha_J}\right)^{(1-\alpha_J)\cdot\frac{\varepsilon_Q-1}{\varepsilon_Q}} \tag{34}$$

Second, take cost-minimizing condition for industry J's intermediate input purchases:

$$\left(\mu_J\right)^{\frac{1}{\varepsilon_Q}} \left(M_J\right)^{\frac{\varepsilon_Q-1}{\varepsilon_Q}} = \mu_J \left(Q_J\right)^{\frac{\varepsilon_Q-1}{\varepsilon_Q}} \left(\frac{P_J^{in}}{P_J}\right)^{1-\varepsilon_Q} \tag{35}$$

And, third, the following equation takes the cost-minimizing choice of the capital-labor aggregate.

$$\begin{aligned}
\left(1-\mu_J\right)^{\frac{1}{\varepsilon_Q}} \left(\left(\frac{K_J}{\alpha_J}\right)^{\alpha_J} \left(\frac{L_J}{1-\alpha_J}\right)^{1-\alpha_J}\right)^{\frac{\varepsilon_Q-1}{\varepsilon_Q}} &= \left(1-\mu_J\right) \left(Q_J\right)^{\frac{\varepsilon_Q-1}{\varepsilon_Q}} \times \\
&\quad \left(\frac{\left(\frac{1-\beta(1-\delta_K)}{\beta}\right)^{\alpha_J} \left[\sum \Gamma_{IJ}^X \left(P_I\right)^{1-\varepsilon_X}\right]^{\alpha_J/(1-\varepsilon_X)}}{P_J}\right)^{1-\varepsilon_Q}
\end{aligned} \tag{36}$$

Plug Equations 34-36 into Equation 33.

$$\left(P_J\right)^{1-\varepsilon_Q} = \left(1-\mu_J\right)\left(\beta^{-1}-\left(1-\delta_K\right)\right)^{\alpha_J\left(1-\varepsilon_Q\right)}\left[\sum_I \Gamma^X_{IJ}\left(P_I\right)^{1-\varepsilon_X}\right]^{\alpha_J\frac{1-\varepsilon_Q}{1-\varepsilon_X}} + \mu_J\left[\sum_I \Gamma^M_{IJ}\left(P_I\right)^{1-\varepsilon_M}\right]^{\frac{1-\varepsilon_Q}{1-\varepsilon_M}} \tag{37}$$

Equation 37 describes and $N \times N$ system of equations for the N steady-state price levels. This completes the first part of the characterization of the steady state.

For the second part, consider the market clearing condition for good J:

$$Q_J = \delta_{C_J} C_J + \sum_{I=1}^{N}\left(M_{J\to I} + X_{J\to I}\right) \tag{38}$$

Below, I will write out the terms on the right-hand-side of Equation 38 in terms of the steady state prices (which have just been solved for):

First, write out the consumption of good J.

$$\frac{\left(1-\beta\left(1-\delta_{C_J}\right)\right)}{\beta}P_J = \left(\xi_J\right)^{\frac{1}{\varepsilon_D}}\cdot\left(\delta_{C_J}\right)^{\frac{\varepsilon_D-1}{\varepsilon_D}}\left(C_J\right)^{-\frac{1}{\varepsilon_D}}\left(\sum_{I=1}^{N}\left(\xi_I\right)^{\frac{1}{\varepsilon_D}}\left(\delta_{C_I}\cdot C_I\right)^{\frac{\varepsilon_D-1}{\varepsilon_D}}\right)^{-1}$$

$$\delta_{C_J} C_J = \xi_J\left(\delta_{C_J}\right)^{\varepsilon_D}\left(\frac{1-\beta\left(1-\delta_{C_J}\right)}{\beta}\right)^{-\varepsilon_D}\left(P_J\right)^{-\varepsilon_D}\bar{C}^{1-\varepsilon_D}, \tag{39}$$

where $\bar{C}$ is the aggregate consumption bundle, defined as the final term in parentheses on the first line raised to the $1/\left(1-\varepsilon_D\right)$ power.

Then write out the intermediate input purchases from industry J to industry I

$$\begin{aligned} M_{J\to I} &= \left(Q_I\mu_I\right)^{\frac{\varepsilon_M}{\varepsilon_Q}}\cdot\left(M_I\right)^{\frac{\varepsilon_Q-\varepsilon_M}{\varepsilon_M}}\Gamma^M_{JI}\cdot\left(\frac{P_I}{P_J}\right)^{\varepsilon_M} \\ &= Q_I\mu_I\Gamma^M_{JI}\left(P_J\right)^{-\varepsilon_M}\left(P_I^{in}\right)^{\varepsilon_M-\varepsilon_Q}\left(P_I\right)^{\varepsilon_Q} \\ &= Q_I\mu_I\Gamma^M_{JI}\left(P_J\right)^{-\varepsilon_M}\left(\sum_{J'}\Gamma^M_{J'I}\left(P_{J'}\right)^{1-\varepsilon_M}\right)^{\frac{\varepsilon_M-\varepsilon_Q}{1-\varepsilon_M}}\left(P_I\right)^{\varepsilon_Q} \end{aligned} \tag{40}$$

And, finally, write out the investment input purchases from industry J sold to industry I. Begin by writing out the total investment purchases of industry J.

$$\left(\frac{K_J}{\alpha_J}\right) = \left(\frac{1-\beta(1-\delta_K)}{\beta}\left[\sum\Gamma^X_{IJ}\left(P_I\right)^{1-\varepsilon_X}\right]^{\frac{1}{1-\varepsilon_X}}\right)^{-1+\alpha_J\left(1-\varepsilon_Q\right)}\left(1-\mu_J\right)Q_J\left(P_J\right)^{\varepsilon_Q}$$

$$X_J = \left(1-\mu_J\right)Q_J\alpha_J\delta_K\left(\frac{1-\beta(1-\delta_K)}{\beta}\left[\sum\Gamma^X_{IJ}\left(P_I\right)^{1-\varepsilon_X}\right]^{\frac{1}{1-\varepsilon_X}}\right)^{-1+\alpha_J\left(1-\varepsilon_Q\right)}\left(P_J\right)^{\varepsilon_Q}$$

So:

$$
\begin{aligned}
X_{J\to I} &= X_I \cdot \Gamma_{JI}^{X} \cdot \left(\frac{P_J}{P_I^{inv}}\right)^{-\varepsilon_X} \\
&= X_I \cdot \Gamma_{JI}^{X} \cdot \left(P_J\right)^{-\varepsilon_X} \left(P_I^{inv}\right)^{\varepsilon_X} \\
&= Q_I \left(1-\mu_I\right) \alpha_I \delta_K \left(\frac{1-\beta(1-\delta_K)}{\beta}\right)^{-1+\alpha_I\left(1-\varepsilon_Q\right)} \Gamma_{JI}^{X} \times \\
&\left[\sum_{J'} \Gamma_{J'I}^{X} \left(P_{J'}\right)^{1-\varepsilon_X}\right]^{\frac{\varepsilon_X-1+\alpha_I\left(1-\varepsilon_Q\right)}{1-\varepsilon_X}} \left(P_J\right)^{-\varepsilon_X} \left(P_I\right)^{\varepsilon_Q} \qquad (41)
\end{aligned}
$$

Plug in the expressions (Equations 39-41) into the market clearing condition (Equation 38):

$$
Q_J - \sum_{I=1}^{N} \tilde{\Gamma}_{JI} Q_I = \xi_J \left(\delta_{C_J}\right)^{\varepsilon_D} \left(\frac{1-\beta\left(1-\delta_{C_J}\right)}{\beta}\right)^{-\varepsilon_D} \left(P_J\right)^{-\varepsilon_D} \bar{C}^{1-\varepsilon_D}
$$

where

$$
\begin{aligned}
\tilde{\Gamma}_{JI} = \left(P_I\right)^{\varepsilon_Q} \times \Bigg\{ & \mu_I \Gamma_{JI}^{M} \left[\sum_{J'} \Gamma_{J'I}^{M} \left(P_{J'}\right)^{1-\varepsilon_M}\right]^{\frac{\varepsilon_M-\varepsilon_Q}{1-\varepsilon_M}} \left(P_J\right)^{-\varepsilon_M} \\
& + \left(1-\mu_I\right) \alpha_I \delta_K \left(\frac{1-\beta(1-\delta_K)}{\beta}\right)^{-1+\alpha_I\left(1-\varepsilon_Q\right)} \Gamma_{JI}^{X} \left[\sum_{J'} \Gamma_{J'I}^{X} \left(P_{J'}\right)^{1-\varepsilon_X}\right]^{-1+\alpha_I \frac{1-\varepsilon_Q}{1-\varepsilon_X}} \left(P_J\right)^{-\varepsilon_X} \Bigg\}
\end{aligned}
$$

We can solve for the Q vector using linear algebra. From here, we can solve for the steady state shares:

$$L_J = Q_J \left(1 - \alpha_J\right)\left(1 - \mu_J\right)\left(P_J\right)^{\varepsilon_Q}\left(\frac{1 - \beta\left(1 - \delta_K\right)}{\beta} P_J^{inv}\right)^{\alpha_J\left(1 - \varepsilon_Q\right)} \tag{42}$$

$$C_J = \xi_J \delta_{C_J}^{\varepsilon_D - 1} \bar{C}^{1 - \varepsilon_D}\left(\frac{1 - \beta\left(1 - \delta_{C_J}\right)}{\beta}\right)^{-\varepsilon_D}\left(P_J\right)^{-\varepsilon_D}$$

$$S_I^C = \frac{\left(\xi_I\right)^{\frac{1}{\varepsilon_D}}\left(\delta_{C_I} \cdot C_I\right)^{\frac{\varepsilon_D - 1}{\varepsilon_D}}}{\sum\left(\xi_{I'}\right)^{\frac{1}{\varepsilon_D}}\left(\delta_{C_{I'}} C_{I'}\right)^{\frac{\varepsilon_D - 1}{\varepsilon_D}}} \tag{43}$$

$$\frac{M_{J \to I}}{Q_J} = \left(Q_J\right)^{-1} Q_I \mu_I \Gamma_{JI}^M \left(P_I^{in}\right)^{\varepsilon_M - \varepsilon_Q}\left(P_J\right)^{-\varepsilon_M}\left(P_I\right)^{\varepsilon_Q}$$

$$\frac{X_{J \to I}}{Q_J} = \left(Q_J\right)^{-1} Q_I \left(1 - \mu_I\right)\alpha_I \delta_K \left(\frac{1 - \beta(1 - \delta_K)}{\beta}\right)^{-1 + \alpha_I\left(1 - \varepsilon_Q\right)} \Gamma_{JI}^X \times$$

$$\left[\sum_{J'} \Gamma_{J'I}^X \left(P_{J'}\right)^{1 - \varepsilon_X}\right]^{\frac{\varepsilon_X - 1 + \alpha_J\left(1 - \varepsilon_Q\right)}{1 - \varepsilon_X}}\left(P_J\right)^{-\varepsilon_X}\left(P_I\right)^{\varepsilon_Q}$$

$$\left[S_X^1\right]_{IJ} = \Gamma_{IJ}^X \left(\frac{P_J^{inv}}{P_I}\right)^{\varepsilon_X - 1} \tag{44}$$

$$\left[S_M^1\right]_{IJ} = \Gamma_{IJ}^M \left(\frac{P_J^{in}}{P_I}\right)^{\varepsilon_M - 1} \tag{45}$$

Clearly, these equations depend on Q_J and the steady-state prices. Note that, however, these figures have already been solved for. For future reference, define $\tilde{S}_M^Q$ as the matrix that has, in its J, I entry, the fraction of good J that is sold to industry I as an intermediate input: $\left[\tilde{S}_M^Q\right]_{JI} \equiv \frac{M_{J \to I}}{Q_J}$. Similarly, define $\left[\tilde{S}_X^Q\right]_{JI} \equiv \frac{X_{J \to I}}{Q_J}$. Equation 42 characterizes the share of labor that is employed in industry J, in the steady state. Use $\tilde{S}^L$ as the $N \times N$ matrix that has, in its J^{th} column, this steady-state share. Also for future reference, define $\tilde{S}_I^C$ as the matrix that has S_I^C (as given in Equation 43) in its I^{th} column. And, finally, use $\left[\tilde{S}_C^Q\right]_J$ to denote the share of good J that is consumed (which can be computed by subtracting the sum of the $\left[\tilde{S}_M^Q\right]_{IJ}$ and $\left[\tilde{S}_X^Q\right]_{IJ}$ from 1.)

F.2 Log linearization

The log linearization of the first order conditions are rather straightforward to derive. Below, I will derive Equations 46 and 47. In all of these equations, a lower-case letter with the circumflex ($\hat{}$) denotes log-deviation from the steady state.

$$\begin{aligned}
\hat{x}_{tJ} &= \delta_K^{-1}\hat{k}_{t+1,J} + (1-\delta_K^{-1})\hat{k}_{tJ} \\
\hat{q}_{tJ} &= \delta_{C_J}^{-1} S_{CJ}^{Q} \hat{c}_{t+1J} + \left(1-\delta_{C_J}^{-1}\right) S_{CJ}^{Q}\hat{c}_{tJ} \qquad (46) \\
&\quad + \sum_{I=1}^{N}\left(S_{M,J\to I}^{Q}\hat{m}_{tJ\to I} + S_{X,J\to I}^{Q}\hat{x}_{tJ\to I}\right) \\
\frac{1}{1-\beta\left(1-\delta_{C_J}\right)}\hat{p}_{tJ} - \frac{\beta\left(1-\delta_{C_J}\right)}{1-\beta\left(1-\delta_{C_J}\right)}\hat{p}_{t+1,J} &\approx -\frac{1}{\varepsilon_D}\hat{c}_{t+1J} \qquad (47) \\
&\quad - \sum_{I}\frac{\left(\xi_I\right)^{\frac{1}{\varepsilon_D}}\left(\delta_{C_I}\cdot C_I\right)^{\frac{\varepsilon_D-1}{\varepsilon_D}}}{\sum_{I'}\left(\xi_{I'}\right)^{\frac{1}{\varepsilon_D}}\left(\delta_{C_{I'}}C_{I'}\right)^{\frac{\varepsilon_D-1}{\varepsilon_D}}}\left[\frac{\varepsilon_D-1}{\varepsilon_D}\hat{c}_{t+1,I}\right] \\
\hat{p}_{tI} - \hat{p}_{tJ} &= \frac{\varepsilon_Q-1}{\varepsilon_Q}\hat{a}_{tJ} + \frac{1}{\varepsilon_Q}\hat{q}_{tJ} + \left(\frac{1}{\varepsilon_M}-\frac{1}{\varepsilon_Q}\right)\hat{m}_{tJ} - \frac{1}{\varepsilon_M}\hat{m}_{t,I\to J} \\
\hat{p}_{tI} &= \hat{p}_{tJ}^{inv} + \frac{1}{\varepsilon_X}\left(\hat{x}_{tJ} - \hat{x}_{t,I\to J}\right) \\
\frac{1}{\varepsilon_{LS}}\sum_{J'=1}^{N} S_J^L \hat{l}_{tJ} &= \hat{p}_{tJ} + \frac{\varepsilon_Q-1}{\varepsilon_Q}\hat{a}_{tJ} + \frac{\left(\varepsilon_Q-1\right)\left(1-\alpha_J\right)}{\varepsilon_Q}\hat{b}_{tJ} \\
&\quad + \frac{1}{\varepsilon_Q}\hat{q}_{tJ} + \alpha_J\frac{\varepsilon_Q-1}{\varepsilon_Q}\hat{k}_{tJ} + \frac{\alpha_J - 1 - \alpha_J\cdot\varepsilon_Q}{\varepsilon_Q}\hat{l}_{tJ} \\
\frac{1}{1-\beta\left(1-\delta_K\right)}\hat{p}_{tJ}^{inv} - \frac{\beta\left(1-\delta_K\right)}{1-\beta\left(1-\delta_K\right)}\hat{p}_{t+1J}^{inv} &= \hat{p}_{t+1,J} + \frac{1}{\varepsilon_Q}\hat{q}_{t+1,J} \\
&\quad + \frac{\varepsilon_Q-1}{\varepsilon_Q}\hat{a}_{t+1,J} + \frac{\left(\varepsilon_Q-1\right)\left(1-\alpha_J\right)}{\varepsilon_Q}\hat{b}_{t+1,J} \\
&\quad + \frac{\left(\varepsilon_Q-1\right)\left(1-\alpha_J\right)}{\varepsilon_Q}\hat{l}_{t+1,J} + \left[-1+\alpha_J\cdot\frac{\varepsilon_Q-1}{\varepsilon_Q}\right]\hat{k}_{t+1,J} \\
\hat{q}_{tJ} &= \hat{a}_{tJ} + \alpha_J\left(1-S_{M_J}\right)\hat{k}_{tJ} + \left(1-\alpha_J\right)\left(1-S_{M_J}\right)\hat{b}_{tJ} \\
&\quad + \left(1-\alpha_J\right)\left(1-S_{M_J}\right)\hat{l}_{tJ} + S_{M_J}\hat{m}_{tJ}
\end{aligned}$$

To derive Equation 46, take the market clearing condition for good J,

$$\begin{aligned}
\log\left[\exp\hat{q}_{tJ}\right] &= \log\left[-\left(1-\delta_{C_J}\right)S_{CJ}^{Q}\exp\hat{c}_{t,J} + S_{CJ}^{Q}\exp\hat{c}_{t+1,J} + \sum_{I=1}^{N} S_{M,J\to I}^{Q}\exp\hat{m}_{tJ\to I} + S_{X,J\to I}^{Q}\exp\hat{x}_{tJ\to I}\right] \\
&\approx S_{CJ}^{Q}\delta_{C_J}^{-1}\hat{c}_{t+1,J} + S_{CJ}^{Q}\left(1-\delta_{C_J}^{-1}\right)\hat{c}_{tJ} + \sum_{I=1}^{N} S_{M,J\to I}^{Q}\hat{m}_{tJ\to I} + S_{X,J\to I}^{Q}\hat{x}_{tJ\to I}
\end{aligned}$$

The following set of calculations yield Equation 47:

$$P_J\left[\frac{1}{\beta}\frac{P_{t-1,J}}{P_J}-\frac{P_{tJ}}{P_J}\left(1-\delta_{C_J}\right)\right]=\left(\xi_J\right)^{\frac{1}{\varepsilon_D}}\left(\delta_{C_J}\right)^{\frac{\varepsilon_D-1}{\varepsilon_D}}\left(C_J\right)^{-\frac{1}{\varepsilon_D}}\left(\exp\left\{\hat{c}_{tJ}\right\}\right)^{-\frac{1}{\varepsilon_D}}\times$$
$$\left(\sum_{I=1}^{N}\left(\xi_I\right)^{\frac{1}{\varepsilon_D}}\left(\delta_{C_I}\cdot C_{tI}\right)^{\frac{\varepsilon_D-1}{\varepsilon_D}}\right)^{-1}$$

$$\frac{1}{1-\beta\left(1-\delta_{C_J}\right)}\exp\hat{p}_{t-1,J}-\frac{\beta\left(1-\delta_{C_J}\right)}{1-\beta\left(1-\delta_{C_J}\right)}\exp\hat{p}_{tJ}=\left(\exp\hat{c}_{tJ}\right)^{-\frac{1}{\varepsilon_D}}\times$$
$$\left(\sum_{I=1}^{N}\frac{\left(\xi_I\right)^{\frac{1}{\varepsilon_D}}\left(\delta_{C_I}\cdot C_I\right)^{\frac{\varepsilon_D-1}{\varepsilon_D}}}{\sum\left(\xi_{I'}\right)^{\frac{1}{\varepsilon_D}}\left(\delta_{C_{I'}}C_{I'}\right)^{\frac{\varepsilon_D-1}{\varepsilon_D}}}\exp\left\{\hat{c}_{tI}\right\}^{\frac{\varepsilon_D-1}{\varepsilon_D}}\right)^{-1}$$

$$\frac{1}{1-\beta\left(1-\delta_{C_J}\right)}\hat{p}_{t-1J}-\frac{\beta\left(1-\delta_{C_J}\right)}{1-\beta\left(1-\delta_{C_J}\right)}\hat{p}_{tJ}\approx-\frac{1}{\varepsilon_D}\hat{c}_{tJ}$$
$$-\sum\frac{\left(\xi_I\right)^{\frac{1}{\varepsilon_D}}\left(\delta_{C_I}\cdot C_I\right)^{\frac{\varepsilon_D-1}{\varepsilon_D}}}{\sum\left(\xi_{I'}\right)^{\frac{1}{\varepsilon_D}}\left(\delta_{C_{I'}}C_{I'}\right)^{\frac{\varepsilon_D-1}{\varepsilon_D}}}\left[\frac{\varepsilon_D-1}{\varepsilon_D}\hat{c}_{tI}\right]$$

Write the log-linearized equations, as given in the beginning of the subsection, in matrix form.

$$\begin{aligned}
\hat{k}_{t+1}&=\delta_K\hat{\mathbb{X}}_t+\left(1-\delta_K\right)\hat{k}_t\\
\hat{q}_t&=\delta_C^{-1}\tilde{S}_C^Q\hat{c}_{t+1}+\left(\mathbf{I}-\delta_C^{-1}\right)\tilde{S}_C^Q\hat{c}_t+\tilde{S_X^Q}\hat{x}_t+\tilde{S}_M^Q\hat{m}_t\\
\hat{p}_t&=\beta\left(\mathbf{I}-\delta_C\right)\hat{p}_{t+1}-\frac{1}{\varepsilon_D}\left(\mathbf{I}-\beta\left(\mathbf{I}-\delta_C\right)\right)\left[\mathbf{I}+S_I^C\left(\varepsilon_D-1\right)\right]\hat{c}_{t+1}\\
\hat{m}_t&=\frac{\varepsilon_M}{\varepsilon_Q}\left(\varepsilon_Q-1\right)T_1\hat{a}_t+\frac{\varepsilon_M}{\varepsilon_Q}T_1\hat{q}_t+\left(1-\frac{\varepsilon_M}{\varepsilon_Q}\right)T_1\hat{\mathbb{M}}_t+\varepsilon_M\left[T_1-T_2\right]\hat{p}_t\\
\hat{x}_t&=T_1\hat{\mathbb{X}}_t+\varepsilon_X T_1\hat{p}_t^{inv}-\varepsilon_X T_2\hat{p}_t\\
\frac{1}{\varepsilon_{LS}}S^L\hat{l}_t&=\hat{p}_t+\frac{\varepsilon_Q-1}{\varepsilon_Q}\hat{a}_t+\frac{\left(\varepsilon_Q-1\right)\left(\mathbf{I}-\alpha\right)}{\varepsilon_Q}\hat{b}_t+\frac{1}{\varepsilon_Q}\hat{q}_t+\frac{\varepsilon_Q-1}{\varepsilon_Q}\alpha\hat{k}_t+\frac{\alpha-\mathbf{I}-\alpha\varepsilon_Q}{\varepsilon_Q}\hat{l}_t\\
\hat{p}_t^{inv}&=\beta(1-\delta_K)\hat{p}_{t+1}^{inv}+\left(1-\beta(1-\delta_K)\right)\left[\hat{p}_{t+1}+\frac{1}{\varepsilon_Q}\hat{q}_{t+1}+\frac{\varepsilon_Q-1}{\varepsilon_Q}\hat{a}_{t+1}\right.\\
&\quad\left.+\left(-\mathbf{I}+\alpha\frac{\varepsilon_Q-1}{\varepsilon_Q}\right)\hat{k}_{t+1}+\left(\mathbf{I}-\alpha\right)\frac{\varepsilon_Q-1}{\varepsilon_Q}\left(\hat{l}_{t+1}+\hat{b}_{t+1}\right)\right]\\
\hat{q}_t&=\hat{a}_t+\left(\mathbf{I}-\alpha\right)\left(\mathbf{I}-S_M\right)\hat{b}_t+\alpha\left(\mathbf{I}-S_M\right)\hat{k}_t+\left(\mathbf{I}-\alpha\right)\left(\mathbf{I}-S_M\right)\hat{l}_t+S_M\hat{\mathbb{M}}_t
\end{aligned}$$

In these equations T_1 refers to the $N^2\times N$ matrix equal to $\mathbf{1}\otimes\mathbf{I}$, where $\mathbf{1}$ is an $N\times 1$ vector of 1s and $\otimes$ is the Kronecker product. Similarly, T_2 equals $\mathbf{I}\otimes\mathbf{1}$. Also, S_M is a diagonal matrix with the steady-state intermediate cost shares along the diagonal; δ_C is a matrix with δ_{C_J}s along the diagonal; and α is a diagonal matrix with the α_Js along the diagonal. Finally, $\hat{\mathbb{M}}_t$ and $\hat{\mathbb{X}}_t$ are the $N\times 1$ vectors which contain the intermediate input bundles and investment input bundles employed by each industry, whereas $\hat{m}_t$ and $\hat{x}_t$ refer to the $N^2 \times 1$ vectors which contain the flows of intermediate and investment inputs across pairs of industries.

F.3 System reduction

Step 1: Substitute out $\hat{x}_t$ and $\hat{m}_t$:

$$\begin{aligned}
\hat{m}_t &= \frac{\varepsilon_M}{\varepsilon_Q}(\varepsilon_Q - 1) T_1 \hat{a}_t + \frac{\varepsilon_M}{\varepsilon_Q} T_1 \hat{q}_t + \left(1 - \frac{\varepsilon_M}{\varepsilon_Q}\right) T_1 \hat{\mathbb{M}}_t + \varepsilon_M [T_1 - T_2] \hat{p}_t \\
\hat{x}_t &= T_1 \hat{\mathbb{X}}_t + \varepsilon_X T_1 \hat{p}_t^{inv} - \varepsilon_X T_2 \hat{p}_t
\end{aligned}$$

to get:

$$\begin{aligned}
\hat{k}_{t+1} &= \delta_K \hat{\mathbb{X}}_t + (1 - \delta_K) \hat{k}_t \\
\left(\mathbf{I} - \frac{\varepsilon_M}{\varepsilon_Q} \tilde{S}_M^Q T_1\right) \hat{q}_t &= \delta_C^{-1} \tilde{S}_C^Q \hat{c}_{t+1} + \left(\mathbf{I} - \delta_C^{-1}\right) \tilde{S}_C^Q \hat{c}_t + \tilde{S_X^Q} T_1 \hat{\mathbb{X}}_t + \frac{\varepsilon_M}{\varepsilon_Q}(\varepsilon_Q - 1) \tilde{S}_M^Q T_1 \hat{a}_t \\
&+ \left(1 - \frac{\varepsilon_M}{\varepsilon_Q}\right) \tilde{S}_M^Q T_1 \hat{\mathbb{M}}_t + \varepsilon_X \tilde{S_X^Q} T_1 \hat{p}_t^{inv} + \left[\varepsilon_M \tilde{S}_M^Q [T_1 - T_2] - \varepsilon_X \tilde{S_X^Q} T_2\right] \hat{p}_t \\
\hat{p}_t &= \beta (\mathbf{I} - \delta_C) \hat{p}_{t+1} - \frac{1}{\varepsilon_D} (\mathbf{I} - \beta (\mathbf{I} - \delta_C)) \left[\mathbf{I} + S_I^C (\varepsilon_D - 1)\right] \hat{c}_{t+1} \\
\frac{1}{\varepsilon_{LS}} S^L \hat{l}_t &= \hat{p}_t + \frac{\varepsilon_Q - 1}{\varepsilon_Q} \hat{a}_t + \frac{(\varepsilon_Q - 1)(\mathbf{I} - \alpha)}{\varepsilon_Q} \hat{b}_t + \frac{1}{\varepsilon_Q} \hat{q}_t + \frac{\varepsilon_Q - 1}{\varepsilon_Q} \alpha \hat{k}_t + \frac{\alpha - \mathbf{I} - \alpha \varepsilon_Q}{\varepsilon_Q} \hat{l}_t \\
\hat{p}_t^{inv} &= \beta(1 - \delta_K) \hat{p}_{t+1}^{inv} + (1 - \beta(1 - \delta_K)) \left[\hat{p}_{t+1} + \frac{1}{\varepsilon_Q} \hat{q}_{t+1} + \frac{\varepsilon_Q - 1}{\varepsilon_Q} \hat{a}_{t+1}\right. \\
&+ \left.\left(-\mathbf{I} + \alpha \frac{\varepsilon_Q - 1}{\varepsilon_Q}\right) \hat{k}_{t+1} + (\mathbf{I} - \alpha) \frac{\varepsilon_Q - 1}{\varepsilon_Q} \left(\hat{l}_{t+1} + \hat{b}_{t+1}\right)\right] \\
\hat{q}_t &= \hat{a}_t + (\mathbf{I} - \alpha)(\mathbf{I} - S_M) \hat{b}_t + \alpha (\mathbf{I} - S_M) \hat{k}_t + (\mathbf{I} - \alpha)(\mathbf{I} - S_M) \hat{l}_t + S_M \hat{\mathbb{M}}_t
\end{aligned}$$

Step 2: Use $S_1^X \hat{p}_t = \hat{p}_t^{inv}$ (S_1^X is the matrix that gives the share of different industries' outputs in the investment input bundle) and $\hat{\mathbb{X}}_t = \delta_K^{-1} \hat{k}_{t+1} + \left(1 - \delta_K^{-1}\right) \hat{k}_t$; and define $\tilde{\beta} \equiv 1 - \beta (1 - \delta_K)$

to get:

$$\begin{aligned}
\left(\mathbf{I} - \frac{\varepsilon_M}{\varepsilon_Q}\tilde{S}^Q_M T_1\right)\hat{q}_t &= \delta_C^{-1}\tilde{S}^Q_C\hat{c}_{t+1} + \left(\mathbf{I} - \delta_C^{-1}\right)\tilde{S}^Q_C\hat{c}_t + \frac{\varepsilon_M}{\varepsilon_Q}\left(\varepsilon_Q - 1\right)\tilde{S}^Q_M T_1\hat{a}_t + \left(1 - \frac{\varepsilon_M}{\varepsilon_Q}\right)\tilde{S}^Q_M T_1\hat{\mathbb{M}}_t \\
&\quad + \left[\varepsilon_X\tilde{S^Q_X}T_1 S^X_1 + \varepsilon_M\tilde{S}^Q_M\left[T_1 - T_2\right] - \varepsilon_X\tilde{S^Q_X}T_2\right]\hat{p}_t + \tilde{S^Q_X}T_1\delta_K^{-1}\hat{k}_{t+1} + \tilde{S^Q_X}T_1\left(1 - \delta_K^{-1}\right)\hat{k}_t \\
\hat{p}_t &= \beta\left(\mathbf{I} - \delta_C\right)\hat{p}_{t+1} - \frac{1}{\varepsilon_D}\left(\mathbf{I} - \beta\left(\mathbf{I} - \delta_C\right)\right)\left[\mathbf{I} + S^C_I\left(\varepsilon_D - 1\right)\right]\hat{c}_{t+1} \\
\frac{1}{\varepsilon_{LS}}S^L\hat{l}_t &= \hat{p}_t + \frac{\varepsilon_Q - 1}{\varepsilon_Q}\hat{a}_t + \frac{\left(\varepsilon_Q - 1\right)\left(\mathbf{I} - \alpha\right)}{\varepsilon_Q}\hat{b}_t + \frac{1}{\varepsilon_Q}\hat{q}_t + \frac{\varepsilon_Q - 1}{\varepsilon_Q}\alpha\hat{k}_t + \frac{\alpha - \mathbf{I} - \alpha\varepsilon_Q}{\varepsilon_Q}\hat{l}_t \\
S^X_1\hat{p}_t &= \left[\tilde{\beta}\mathbf{I} + \beta(1 - \delta_K)S^X_1\right]\hat{p}_{t+1} + \tilde{\beta}\left[\frac{1}{\varepsilon_Q}\hat{q}_{t+1} + \frac{\varepsilon_Q - 1}{\varepsilon_Q}\hat{a}_{t+1}\right. \\
&\quad \left. + \left(-\mathbf{I} + \alpha\frac{\varepsilon_Q - 1}{\varepsilon_Q}\right)\hat{k}_{t+1} + \left(\mathbf{I} - \alpha\right)\frac{\varepsilon_Q - 1}{\varepsilon_Q}\left(\hat{l}_{t+1} + \hat{b}_{t+1}\right)\right] \\
\hat{q}_t &= \hat{a}_t + \left(\mathbf{I} - \alpha\right)\left(\mathbf{I} - S_M\right)\hat{b}_t + \alpha\left(\mathbf{I} - S_M\right)\hat{k}_t + \left(\mathbf{I} - \alpha\right)\left(\mathbf{I} - S_M\right)\hat{l}_t + S_M\hat{\mathbb{M}}_t
\end{aligned}$$

Step 3: Use

$$\begin{aligned}
\hat{\mathbb{M}}_t &= \left(\varepsilon_Q - 1\right)\hat{a}_t + \hat{q}_t + \varepsilon_Q\left(\mathbf{I} - S^M_1\right)\hat{p}_t \\
\text{where } S^M_1\hat{p}_t &= \hat{p}^{in}_t
\end{aligned}$$

$$\begin{aligned}
\left(\mathbf{I} - \tilde{S}^Q_M T_1\right)\hat{q}_t &= \delta_C^{-1}\tilde{S}^Q_C\hat{c}_{t+1} + \left(\mathbf{I} - \delta_C^{-1}\right)\tilde{S}^Q_C\hat{c}_t + \left(\varepsilon_Q - 1\right)\tilde{S}^Q_M T_1\hat{a}_t \\
&\quad + \tilde{S^Q_X}T_1\delta_K^{-1}\hat{k}_{t+1} + \tilde{S^Q_X}T_1\left(1 - \delta_K^{-1}\right)\hat{k}_t \\
&\quad + \left[\varepsilon_Q\tilde{S}^Q_M T_1\left(\mathbf{I} - S^M_1\right) + \varepsilon_M\tilde{S}^Q_M\left[T_1 S^M_1 - T_2\right] + \varepsilon_X\tilde{S^Q_X}\left[T_1 S^X_1 - T_2\right]\right]\hat{p}_t \\
\hat{p}_t &= \beta\left(\mathbf{I} - \delta_C\right)\hat{p}_{t+1} - \frac{1}{\varepsilon_D}\left(\mathbf{I} - \beta\left(\mathbf{I} - \delta_C\right)\right)\left[\mathbf{I} + S^C_I\left(\varepsilon_D - 1\right)\right]\hat{c}_{t+1} \\
\frac{1}{\varepsilon_{LS}}S^L\hat{l}_t &= \hat{p}_t + \frac{\varepsilon_Q - 1}{\varepsilon_Q}\hat{a}_t + \frac{\left(\varepsilon_Q - 1\right)\left(\mathbf{I} - \alpha\right)}{\varepsilon_Q}\hat{b}_t + \frac{1}{\varepsilon_Q}\hat{q}_t + \frac{\varepsilon_Q - 1}{\varepsilon_Q}\alpha\hat{k}_t + \frac{\alpha - \mathbf{I} - \alpha\varepsilon_Q}{\varepsilon_Q}\hat{l}_t \qquad (48) \\
S^X_1\hat{p}_t &= \left[\tilde{\beta}\mathbf{I} + \beta(1 - \delta_K)S^X_1\right]\hat{p}_{t+1} + \tilde{\beta}\frac{1}{\varepsilon_Q}\hat{q}_{t+1} \\
&\quad + \tilde{\beta}\frac{\varepsilon_Q - 1}{\varepsilon_Q}\hat{a}_{t+1} + \tilde{\beta}\left(-\mathbf{I} + \alpha\frac{\varepsilon_Q - 1}{\varepsilon_Q}\right)\hat{k}_{t+1} + \tilde{\beta}\left(\mathbf{I} - \alpha\right)\frac{\varepsilon_Q - 1}{\varepsilon_Q}\left(\hat{l}_{t+1} + \hat{b}_{t+1}\right) \\
\hat{q}_t &= \left(\mathbf{I} - S_M\right)^{-1}\left(\mathbf{I} + S_M\left(\varepsilon_Q - 1\right)\right)\hat{a}_t + \left(\mathbf{I} - \alpha\right)\hat{b}_t + \alpha\hat{k}_t \qquad (49) \\
&\quad + \left(\mathbf{I} - \alpha\right)\hat{l}_t + \left(\mathbf{I} - S_M\right)^{-1}S_M\varepsilon_Q\left(\mathbf{I} - S^M_1\right)\hat{p}_t
\end{aligned}$$

Step 4: Use the production function, given in Equation 49, to substitute $\hat{q}_t$ out of the first, third, and fourth equations:

$$
\begin{aligned}
\frac{1}{\varepsilon_Q}\hat{q}_t &= \frac{1}{\varepsilon_Q}\left(\mathbf{I}-S_M\right)^{-1}\left(\mathbf{I}+S_M\left(\varepsilon_Q-1\right)\right)\hat{a}_t+\frac{1}{\varepsilon_Q}\left(\mathbf{I}-\alpha\right)\hat{b}_t+\frac{1}{\varepsilon_Q}\alpha\hat{k}_t \qquad (50)\\
&+\frac{1}{\varepsilon_Q}\left(\mathbf{I}-\alpha\right)\hat{l}_t+\left(\mathbf{I}-S_M\right)^{-1}S_M\left(\mathbf{I}-S_1^M\right)\hat{p}_t\\
\left(\mathbf{I}-\tilde{S}_M^QT_1\right)\hat{q}_t &= \left(\mathbf{I}-\tilde{S}_M^QT_1\right)\left(\mathbf{I}-S_M\right)^{-1}\left(\mathbf{I}+S_M\left(\varepsilon_Q-1\right)\right)\hat{a}_t+\left(\mathbf{I}-\tilde{S}_M^QT_1\right)\left(\mathbf{I}-\alpha\right)\hat{b}_t\\
&+\left(\mathbf{I}-\tilde{S}_M^QT_1\right)\alpha\hat{k}_t+\left(\mathbf{I}-\tilde{S}_M^QT_1\right)\left(\mathbf{I}-\alpha\right)\hat{l}_t\\
&+\left(\mathbf{I}-\tilde{S}_M^QT_1\right)\left(\mathbf{I}-S_M\right)^{-1}S_M\left(\mathbf{I}-S_1^M\right)\varepsilon_Q\hat{p}_t
\end{aligned}
$$

to get

$$
\begin{aligned}
0 &= \delta_C^{-1}\tilde{S}_C^Q\hat{c}_{t+1}+\left(\mathbf{I}-\delta_C^{-1}\right)\tilde{S}_C^Q\hat{c}_t\\
&+\left[\left(\varepsilon_Q-1\right)\tilde{S}_M^QT_1-\left(\mathbf{I}-\tilde{S}_M^QT_1\right)\left(\mathbf{I}-S_M\right)^{-1}\left(\mathbf{I}+S_M\left(\varepsilon_Q-1\right)\right)\right]\hat{a}_t\\
&+\tilde{S_X^Q}T_1\delta_K^{-1}\hat{k}_{t+1}+\left[\tilde{S_X^Q}T_1\left(1-\delta_K^{-1}\right)-\left(\mathbf{I}-\tilde{S}_M^QT_1\right)\alpha\right]\hat{k}_t\\
&-\left(\mathbf{I}-\tilde{S}_M^QT_1\right)\left(\mathbf{I}-\alpha\right)\hat{b}_t-\left(\mathbf{I}-\tilde{S}_M^QT_1\right)\left(\mathbf{I}-\alpha\right)\hat{l}_t\\
&+\left[\varepsilon_X\tilde{S_X^Q}\left[T_1S_1^X-T_2\right]-\left(\mathbf{I}-\tilde{S}_M^QT_1\right)\varepsilon_Q\left(\mathbf{I}-S_M\right)^{-1}S_M\left(\mathbf{I}-S_1^M\right)\right]\hat{p}_t\\
&+\left[\varepsilon_Q\tilde{S}_M^QT_1\left(\mathbf{I}-S_1^M\right)+\varepsilon_M\tilde{S}_M^Q\left[T_1S_1^M-T_2\right]\right]\hat{p}_t\\
\hat{p}_t &= \beta\left(\mathbf{I}-\delta_C\right)\hat{p}_{t+1}-\frac{1}{\varepsilon_D}\left(\mathbf{I}-\beta\left(\mathbf{I}-\delta_C\right)\right)\left[\mathbf{I}+S_I^C\left(\varepsilon_D-1\right)\right]\hat{c}_{t+1}\\
S_1^X\hat{p}_t &= \left[\tilde{\beta}\mathbf{I}+\beta(1-\delta_K)S_1^X+\tilde{\beta}\left(I-S_M\right)^{-1}S_M\left(\mathbf{I}-S_1^M\right)\right]\hat{p}_{t+1}\\
&+\tilde{\beta}\left[\frac{1}{\varepsilon_Q}\left(\mathbf{I}-S_M\right)^{-1}\left(\mathbf{I}+S_M\left(\varepsilon_Q-1\right)\right)+\frac{\varepsilon_Q-1}{\varepsilon_Q}\mathbf{I}\right]\hat{a}_{t+1}\\
&+\tilde{\beta}\left(\mathbf{I}-\alpha\right)\hat{b}_{t+1}+\tilde{\beta}\left(-\mathbf{I}+\alpha\right)\hat{k}_{t+1}+\tilde{\beta}\left(\mathbf{I}-\alpha\right)\hat{l}_{t+1}
\end{aligned}
$$

Step 5: Use the following equation:

$$
\begin{aligned}
\left(\mathbf{I}-\alpha\right)\hat{l}_t &= \vartheta\left(\mathbf{I}-\alpha\right)\hat{b}_t+\vartheta\alpha\hat{k}_t+\vartheta\left[\frac{\varepsilon_Q-1}{\varepsilon_Q}\mathbf{I}+\frac{1}{\varepsilon_Q}\left(\mathbf{I}-S_M\right)^{-1}\left(\mathbf{I}+S_M\left(\varepsilon_Q-1\right)\right)\right]\hat{a}_t \qquad (51)\\
&+\vartheta\left[\left(\mathbf{I}-S_M\right)^{-1}S_M\left(\mathbf{I}-S_1^M\right)+\mathbf{I}\right]\hat{p}_t
\end{aligned}
$$

where $\vartheta=\left(\mathbf{I}-\alpha\right)\left[\frac{1}{\varepsilon_{LS}}S^L+\alpha\right]^{-1}$

(this equation comes from plugging Equation 49 into Equation 48 and re-arranging) to get

$$
\begin{aligned}
0 &= \delta_C^{-1} \tilde{S}_C^Q \hat{c}_{t+1} + \left(\mathbf{I} - \delta_C^{-1}\right) \tilde{S}_C^Q \hat{c}_t \\
&+ \left[\left(\varepsilon_Q - 1\right) \tilde{S}_M^Q T_1 - \left(\mathbf{I} - \tilde{S}_M^Q T_1\right) \left(\mathbf{I} + \vartheta \varepsilon_Q^{-1}\right) \left(\mathbf{I} - S_M\right)^{-1} \left(\mathbf{I} + S_M \left(\varepsilon_Q - 1\right)\right)\right] \hat{a}_t \\
&- \left(\mathbf{I} - \tilde{S}_M^Q T_1\right) \vartheta \frac{\varepsilon_Q - 1}{\varepsilon_Q} \hat{a}_t + \tilde{S_X^Q} T_1 \delta_K^{-1} \hat{k}_{t+1} \\
&+ \left[\tilde{S_X^Q} T_1 \left(1 - \delta_K^{-1}\right) - \left(\mathbf{I} - \tilde{S}_M^Q T_1\right) \left(\mathbf{I} + \vartheta\right) \alpha\right] \hat{k}_t - \left(\mathbf{I} - \tilde{S}_M^Q T_1\right) \left(\mathbf{I} + \vartheta\right) \left(\mathbf{I} - \alpha\right) \hat{b}_t \\
&+ \left[\varepsilon_Q \tilde{S}_M^Q T_1 \left(\mathbf{I} - S_1^M\right) + \varepsilon_M \tilde{S}_M^Q \left[T_1 S_1^M - T_2\right] + \varepsilon_X \tilde{S_X^Q} \left[T_1 S_1^X - T_2\right]\right] \hat{p}_t \\
&+ \left[-\left(\mathbf{I} - \tilde{S}_M^Q T_1\right) \left[\varepsilon_Q \left(\mathbf{I} - S_M\right)^{-1} S_M \left(\mathbf{I} - S_1^M\right) + \vartheta \left[\left(\mathbf{I} - S_M\right)^{-1} S_M \left(\mathbf{I} - S_1^M\right) + \mathbf{I}\right]\right]\right] \hat{p}_t \\
\hat{p}_t &= \beta \left(\mathbf{I} - \delta_C\right) \hat{p}_{t+1} - \frac{1}{\varepsilon_D} \left(\mathbf{I} - \beta \left(\mathbf{I} - \delta_C\right)\right) \left[\mathbf{I} + S_I^C \left(\varepsilon_D - 1\right)\right] \hat{c}_{t+1} \\
S_1^X \hat{p}_t &= \left[\beta(1 - \delta_K) S_1^X + \tilde{\beta} \left(\mathbf{I} + \vartheta\right) \left(\mathbf{I} + \left(\mathbf{I} - S_M\right)^{-1} S_M \left(\mathbf{I} - S_1^M\right)\right)\right] \hat{p}_{t+1} \\
&+ \tilde{\beta} \left(\mathbf{I} + \vartheta\right) \left[\frac{1}{\varepsilon_Q} \left(\mathbf{I} - S_M\right)^{-1} \left(\mathbf{I} + S_M \left(\varepsilon_Q - 1\right)\right) + \frac{\varepsilon_Q - 1}{\varepsilon_Q} \mathbf{I}\right] \hat{a}_{t+1} + \tilde{\beta} \left(\mathbf{I} + \vartheta\right) \left(\mathbf{I} - \alpha\right) \hat{b}_{t+1} \\
&+ \tilde{\beta} \left(-\mathbf{I} + \alpha + \vartheta \alpha\right) \hat{k}_{t+1}
\end{aligned}
\tag{52}
$$

So now we are down to three equations and three sets of endogenous unknowns ($\hat{p}_t$, $\hat{k}_t$, and $\hat{c}_t$). How we proceed will depend on whether we allow for consumption to be durable or not.
Case 1: No Durables
Plug

$$\hat{c}_t = -\varepsilon_D \left[\mathbf{I} + S_I^C \left(\varepsilon_D - 1\right)\right]^{-1} \hat{p}_t$$

in to the other two equations, above, to substitute out the $\hat{c}_t$ vector.

$$
\begin{aligned}
0 = &\left[\left(\varepsilon_Q - 1\right) \tilde{S}_M^Q T_1 - \left(\mathbf{I} - \tilde{S}_M^Q T_1\right) \left(\mathbf{I} + \vartheta \varepsilon_Q^{-1}\right) \left(\mathbf{I} - S_M\right)^{-1} \left(\mathbf{I} + S_M \left(\varepsilon_Q - 1\right)\right) - \left(\mathbf{I} - \tilde{S}_M^Q T_1\right) \vartheta \frac{\varepsilon_Q - 1}{\varepsilon_Q}\right] \hat{a}_t \\
&+ \tilde{S_X^Q} T_1 \delta_K^{-1} \hat{k}_{t+1} + \left[\tilde{S_X^Q} T_1 \left(1 - \delta_K^{-1}\right) - \left(\mathbf{I} - \tilde{S}_M^Q T_1\right) \left(\mathbf{I} + \vartheta\right) \alpha\right] \hat{k}_t - \left(\mathbf{I} - \tilde{S}_M^Q T_1\right) \left(\mathbf{I} + \vartheta\right) \left(\mathbf{I} - \alpha\right) \hat{b}_t \\
&+ \left[-\varepsilon_D \tilde{S}_C^Q \left[\mathbf{I} + S_I^C \left(\varepsilon_D - 1\right)\right]^{-1} + \varepsilon_Q \tilde{S}_M^Q T_1 \left(\mathbf{I} - S_1^M\right) + \varepsilon_M \tilde{S}_M^Q \left[T_1 S_1^M - T_2\right] + \varepsilon_X \tilde{S_X^Q} \left[T_1 S_1^X - T_2\right]\right] \hat{p}_t \\
&- \left(\mathbf{I} - \tilde{S}_M^Q T_1\right) \left[\varepsilon_Q \left(\mathbf{I} - S_M\right)^{-1} S_M \left(\mathbf{I} - S_1^M\right) + \vartheta \left[\left(\mathbf{I} - S_M\right)^{-1} S_M \left(\mathbf{I} - S_1^M\right) + \mathbf{I}\right]\right] \hat{p}_t \\
0 = &-S_1^X \hat{p}_t + \left[\beta(1 - \delta_K) S_1^X + \tilde{\beta} \left(\mathbf{I} + \vartheta\right) \left(\mathbf{I} + \left(I - S_M\right)^{-1} S_M \left(\mathbf{I} - S_1^M\right)\right)\right] \hat{p}_{t+1} \\
&+ \tilde{\beta} \left(\mathbf{I} + \vartheta\right) \left[\frac{1}{\varepsilon_Q} \left(I - S_M\right)^{-1} \left(\mathbf{I} + S_M \left(\varepsilon_Q - 1\right)\right) + \frac{\varepsilon_Q - 1}{\varepsilon_Q} \mathbf{I}\right] \hat{a}_{t+1} \\
&+ \tilde{\beta} \left(\mathbf{I} + \vartheta\right) \left(\mathbf{I} - \alpha\right) \hat{b}_{t+1} + \tilde{\beta} \left(-\mathbf{I} + \alpha + \vartheta \alpha\right) \hat{k}_{t+1}
\end{aligned}
$$

Case 2: Durables

Combine the final two equations in the line before "So now..."

$$
\begin{aligned}
&- S_1^X \frac{1}{\varepsilon_D} \left(\mathbf{I} - \beta\left(\mathbf{I} - \delta_C\right)\right) \\
\times \left[\mathbf{I} + S_I^C\left(\varepsilon_D - 1\right)\right] \hat{c}_{t+1} = &\left[\tilde{\beta}\left(\mathbf{I} + \vartheta\right)\left(\mathbf{I} + \left(\mathbf{I} - S_M\right)^{-1} S_M \left(\mathbf{I} - S_1^M\right)\right) + S_1^X \beta\left(\delta_C - \delta_K \mathbf{I}\right)\right] \hat{p}_{t+1} \\
&+ \tilde{\beta}\left(\mathbf{I} + \vartheta\right)\left[\frac{1}{\varepsilon_Q}\left(\mathbf{I} - S_M\right)^{-1}\left(\mathbf{I} + S_M\left(\varepsilon_Q - 1\right)\right) + \frac{\varepsilon_Q - 1}{\varepsilon_Q}\mathbf{I}\right] \hat{a}_{t+1} \\
&+ \tilde{\beta}\left(\mathbf{I} + \vartheta\right)\left(\mathbf{I} - \alpha\right) \hat{b}_{t+1} + \tilde{\beta}\left(-\mathbf{I} + \alpha + \vartheta\alpha\right) \hat{k}_{t+1}
\end{aligned}
$$

to get.

$$
\begin{aligned}
\hat{c}_t = &\tilde{\vartheta}\left[\tilde{\beta}\left(\mathbf{I} + \vartheta\right)\left(\mathbf{I} + \left(\mathbf{I} - S_M\right)^{-1} S_M \left(\mathbf{I} - S_1^M\right)\right) + S_1^X \beta\left(\delta_C - \mathbf{I}\delta_K\right)\right] \hat{p}_t \\
&+ \tilde{\vartheta}\tilde{\beta}\left(\mathbf{I} + \vartheta\right)\left[\frac{1}{\varepsilon_Q}\left(\mathbf{I} - S_M\right)^{-1}\left(\mathbf{I} + S_M\left(\varepsilon_Q - 1\right)\right) + \frac{\varepsilon_Q - 1}{\varepsilon_Q}\mathbf{I}\right] \hat{a}_t \\
&+ \tilde{\vartheta}\tilde{\beta}\left(\mathbf{I} + \vartheta\right)\left(\mathbf{I} - \alpha\right) \hat{b}_t + \tilde{\beta}\tilde{\vartheta}\left(-\mathbf{I} + \alpha + \vartheta\alpha\right) \hat{k}_t
\end{aligned}
$$

where

$$
\tilde{\vartheta} \equiv \left[-S_1^X \frac{1}{\varepsilon_D}\left(\mathbf{I} - \beta\left(\mathbf{I} - \delta_C\right)\right)\left[\mathbf{I} + S_I^C\left(\varepsilon_D - 1\right)\right]\right]^{-1}
$$

Plug this in:

$$
\begin{aligned}
0 = {} & \tilde{S}_C^Q \tilde{\vartheta} \tilde{\beta} \left(\mathbf{I} + \vartheta\right) \left[\frac{1}{\varepsilon_Q} \left(\mathbf{I} - S_M\right)^{-1} \left(\mathbf{I} + S_M \left(\varepsilon_Q - 1\right)\right) + \frac{\varepsilon_Q - 1}{\varepsilon_Q} \mathbf{I}\right] \hat{a}_t \qquad (53) \\
& + \left[\left(\varepsilon_Q - 1\right) \tilde{S}_M^Q T_1 - \left(\mathbf{I} - \tilde{S}_M^Q T_1\right) \left(\mathbf{I} + \vartheta \varepsilon_Q^{-1}\right) \left(\mathbf{I} - S_M\right)^{-1} \left(\mathbf{I} + S_M \left(\varepsilon_Q - 1\right)\right) - \left(\mathbf{I} - \tilde{S}_M^Q T_1\right) \vartheta \frac{\varepsilon_Q - 1}{\varepsilon_Q}\right] \hat{a}_t \\
& + \delta_C^{-1} \tilde{S}_C^Q \tilde{\vartheta} \left[\tilde{\beta} \left(\mathbf{I} + \vartheta\right) \left(\mathbf{I} + \left(\mathbf{I} - S_M\right)^{-1} S_M \left(\mathbf{I} - S_1^M\right)\right) + S_1^X \beta \left(\delta_C - \mathbf{I} \delta_K\right)\right] \hat{p}_{t+1} \\
& + \left(\mathbf{I} - \delta_C^{-1}\right) \tilde{S}_C^Q \tilde{\vartheta} \left[\tilde{\beta} \left(\mathbf{I} + \vartheta\right) \left(\mathbf{I} + \left(\mathbf{I} - S_M\right)^{-1} S_M \left(\mathbf{I} - S_1^M\right)\right) + S_1^X \beta \left(\delta_C - \mathbf{I} \delta_K\right)\right] \hat{p}_t \\
& + \left[\varepsilon_Q \tilde{S}_M^Q T_1 \left(\mathbf{I} - S_1^M\right) + \varepsilon_M \tilde{S}_M^Q \left[T_1 S_1^M - T_2\right] + \varepsilon_X \tilde{S_X^Q} \left[T_1 S_1^X - T_2\right]\right] \hat{p}_t \\
& + \left[-\left(\mathbf{I} - \tilde{S}_M^Q T_1\right) \left[\left(\mathbf{I} - S_M\right)^{-1} S_M \left(\mathbf{I} - S_1^M\right) \varepsilon_Q + \vartheta \left[\left(\mathbf{I} - S_M\right)^{-1} S_M \left(I - S_1^M\right) + \mathbf{I}\right]\right]\right] \hat{p}_t \\
& + \left[\tilde{S_X^Q} T_1 \delta_K^{-1} + \delta_C^{-1} \tilde{S}_C^Q \tilde{\beta} \tilde{\vartheta} \left(-\mathbf{I} + \alpha + \vartheta \alpha\right)\right] \hat{k}_{t+1} \\
& + \left[\tilde{S_X^Q} T_1 \left(1 - \delta_K^{-1}\right) - \left(\mathbf{I} - \tilde{S}_M^Q T_1\right) \left(\mathbf{I} + \vartheta\right) \alpha + \left(\mathbf{I} - \delta_C^{-1}\right) \tilde{S}_C^Q \tilde{\beta} \tilde{\vartheta} \left(-\mathbf{I} + \alpha + \vartheta \alpha\right)\right] \hat{k}_t \\
& + \left[\tilde{S}_C^Q \tilde{\vartheta} \tilde{\beta} \left(\mathbf{I} + \vartheta\right) \left(\mathbf{I} - \alpha\right) - \left(\mathbf{I} - \tilde{S}_M^Q T_1\right) \left(\mathbf{I} + \vartheta\right) \left(\mathbf{I} - \alpha\right)\right] \hat{b}_t \\
0 = {} & -S_1^X \hat{p}_t + \left[\beta (1 - \delta_K) S_1^X + \tilde{\beta} \left(\mathbf{I} + \vartheta\right) \left(\mathbf{I} + \left(\mathbf{I} - S_M\right)^{-1} S_M \left(\mathbf{I} - S_1^M\right)\right)\right] \hat{p}_{t+1} \qquad (54) \\
& + \tilde{\beta} \left(\mathbf{I} + \vartheta\right) \left[\frac{1}{\varepsilon_Q} \left(I - S_M\right)^{-1} \left(\mathbf{I} + S_M \left(\varepsilon_Q - 1\right)\right) + \mathbf{I} \frac{\varepsilon_Q - 1}{\varepsilon_Q}\right] \hat{a}_{t+1} + \tilde{\beta} \left(\mathbf{I} + \vartheta\right) \left(\mathbf{I} - \alpha\right) \hat{b}_{t+1} \\
& + \tilde{\beta} \left(-\mathbf{I} + \alpha + \vartheta \alpha\right) \hat{k}_{t+1}
\end{aligned}
$$

F.4 Blanchard-Kahn

In Equations 53 and 54, we have expressed the reduced system as

$$
\begin{bmatrix} \mathbb{E}_t[\hat{p}_{t+1}] \\ \hat{k}_{t+1} \end{bmatrix} = \mathbf{\Psi} \begin{bmatrix} \hat{p}_t \\ \hat{k}_t \end{bmatrix} + \mathbf{\Phi} \begin{bmatrix} \hat{a}_t \\ \hat{b}_t \end{bmatrix}
$$

Here, $\mathbf{\Psi}$ has N stable and N unstable eigenvalues.

Using a Jordan decomposition, write $\mathbf{\Psi} = \mathbf{V}\mathbf{D}\mathbf{V}^{-1}$ where $\mathbf{D}$ is diagonal and is ordered such that the N explosive eigenvalues are ordered first and the N stable eigenvalues are ordered last. Re-write:

$$
\begin{aligned}
\Upsilon_{t+1} & \equiv \mathbf{V}^{-1} \begin{bmatrix} \mathbb{E}_t[\hat{p}_{t+1}] \\ \hat{k}_{t+1} \end{bmatrix} = \mathbf{D}\mathbf{V}^{-1} \begin{bmatrix} \hat{p}_t \\ \hat{k}_t \end{bmatrix} + \mathbf{V}^{-1}\mathbf{\Phi} \begin{bmatrix} \hat{a}_t \\ \hat{b}_t \end{bmatrix} \\
& \equiv \mathbf{D}\Upsilon_t + \tilde{\mathbf{\Phi}} \begin{bmatrix} \hat{a}_t \\ \hat{b}_t \end{bmatrix}
\end{aligned}
$$

Partition Υ_t into the first $N \times 1$ block, Υ_{1t}, and the lower $N \times 1$ block, Υ_{2t}. Similarly partition $\tilde{\mathbf{\Phi}}$ and $\mathbf{D}$.

$$\Upsilon_{1,t} = \mathbf{D}_1^{-1}\mathbb{E}_t[\Upsilon_{1,t+1}] - \mathbf{D}_1^{-1}\tilde{\mathbf{\Phi}}\begin{bmatrix}\hat{a}_t \\ \hat{b}_t\end{bmatrix}$$

Substitute recursively

$$\Upsilon_{1,t} = -\mathbf{D}_1^{-1}\sum_{s=0}^{\infty}\mathbf{D}_1^{-s}\tilde{\mathbf{\Phi}}_1\begin{bmatrix}\hat{a}_t \\ \hat{b}_t\end{bmatrix} = -\mathbf{D}_1^{-1}(I - \mathbf{D}_1^{-1})^{-1}\tilde{\mathbf{\Phi}}_1\begin{bmatrix}\hat{a}_t \\ \hat{b}_t\end{bmatrix} \tag{55}$$

For $Y_{2,t}$:

$$\Upsilon_{2,t} = \mathbf{D}_2\Upsilon_{2,t-1} + \tilde{\mathbf{\Phi}}_2 \cdot \begin{bmatrix}\hat{a}_t \\ \hat{b}_t\end{bmatrix}$$

Remember that

$$\begin{bmatrix}\Upsilon_{1,t} \\ \Upsilon_{2,t}\end{bmatrix} = \mathbf{V}^{-1}\begin{bmatrix}\hat{p}_t \\ \hat{k}_t\end{bmatrix},$$

and therefore, from Equation 55

$$\begin{aligned}\hat{p}_t &= -(\mathbf{V}_{11}^{-1})^{-1}\mathbf{V}_{12}^{-1}\hat{k}_t + (\mathbf{V}_{11}^{-1})^{-1}\Upsilon_{1t} \\ &= -(\mathbf{V}_{11}^{-1})^{-1}\mathbf{V}_{12}^{-1}\hat{k}_t - (\mathbf{V}_{11}^{-1})^{-1}\mathbf{D}_1^{-1}(\mathbf{I} - \mathbf{D}_1^{-1})^{-1}\tilde{\mathbf{\Phi}}_1\begin{bmatrix}\hat{a}_t \\ \hat{b}_t\end{bmatrix}\end{aligned} \tag{56}$$

The endogenous state evolves as follows:

$$\begin{aligned}\hat{k}_{t+1} &= \mathbf{\Psi}_{22}\hat{k}_t + \mathbf{\Psi}_{21}\hat{p}_t + \mathbf{\Phi}_2\begin{bmatrix}\hat{a}_t \\ \hat{b}_t\end{bmatrix} \\ &= \underbrace{(\mathbf{\Psi}_{22} - \mathbf{\Psi}_{21}(\mathbf{V}_{11}^{-1})^{-1}\mathbf{V}_{12}^{-1})}_{\equiv M_{kk}}\hat{k}_t + \underbrace{\left(-\mathbf{\Psi}_{21}(\mathbf{V}_{11}^{-1})^{-1}\mathbf{D}_1^{-1}(\mathbf{I} - \mathbf{D}_1^{-1})^{-1}\tilde{\Phi}_1 + \mathbf{\Phi}_2\right)}_{\equiv[M_{ka},\, M_{kb}]}\begin{bmatrix}\hat{a}_t \\ \hat{b}_t\end{bmatrix}\end{aligned} \tag{57}$$

For future reference:

$$\hat{p}_t = \mathbf{\Psi}_{21}^{-1}\hat{k}_{t+1} - \mathbf{\Psi}_{21}^{-1}\mathbf{\Psi}_{22}\hat{k}_t - \mathbf{\Psi}_{21}^{-1}\mathbf{\Phi}_2\begin{bmatrix}\hat{a}_t \\ \hat{b}_t\end{bmatrix} \tag{58}$$

F.5 Obtaining the model filter

Combine Equations 50 and 51 to write $\hat{q}_t$ as a function of the exogenous variables, $\hat{k}$, and $\hat{p}$

$$\begin{aligned}\hat{q}_t &= (\mathbf{I} + \vartheta)(\mathbf{I} - \alpha)\hat{b}_t + (\mathbf{I} + \vartheta)\alpha\hat{k}_t \\ &+ \left[\frac{\varepsilon_Q - 1}{\varepsilon_Q}\vartheta + \left(\frac{\vartheta}{\varepsilon_Q} + \mathbf{I}\right)(\mathbf{I} - S_M)^{-1}(\mathbf{I} + S_M(\varepsilon_Q - 1))\right]\hat{a}_t \\ &+ \left[(\vartheta + \varepsilon_Q\mathbf{I})(\mathbf{I} - S_M)^{-1}S_M\left(\mathbf{I} - S_1^M\right) + \vartheta\right]\hat{p}_t\end{aligned} \tag{59}$$

Plug Equation 57 and 58 in so that we may write:

$$\hat{q}_t = \Phi_{kq}\hat{k}_t + \Phi_{bq}\hat{b}_t + \Phi_{aq}\hat{a}_t, \tag{60}$$

where the Φ_{kq}, Φ_{bq}, and Φ_{aq} are matrices that collect the appropriate terms.[40]

So long as Φ_{kq} is invertible, Equation 60 is equivalent to

$$\hat{k}_t = \Phi_{kq}^{-1}\hat{q}_t - \Phi_{kq}^{-1}\Phi_{bq}\hat{b}_t - \Phi_{kq}^{-1}\Phi_{aq}\hat{a}_t$$

Equation 60, one period ahead, is

$$\hat{q}_{t+1} = \Phi_{kq}\hat{k}_{t+1} + \Phi_{bq}\hat{b}_{t+1} + \Phi_{aq}\hat{a}_{t+1}$$

Apply Equation 57 to this previous equation

$$\begin{aligned}
\hat{q}_{t+1} &= \Phi_{bq}\hat{b}_{t+1} + \Phi_{aq}\hat{a}_{t+1} \\
&\quad +\Phi_{kq}\left(M_{kk}\hat{k}_t + M_{ka}\hat{a}_t + M_{kb}\hat{b}_t\right) \\
&= \Phi_{bq}\hat{b}_{t+1} + \Phi_{aq}\hat{a}_{t+1} \\
&\quad +\Phi_{kq}M_{ka}\hat{a}_t + \Phi_{kq}M_{kb}\hat{b}_t \\
&\quad +\Phi_{kq}M_{kk}\Phi_{kq}^{-1}\hat{q}_t - \Phi_{kq}M_{kk}\Phi_{kq}^{-1}\Phi_{bq}\hat{b}_t - \Phi_{kq}M_{kk}\Phi_{kq}^{-1}\Phi_{aq}\hat{a}_t \\
&= \Phi_{bq}\hat{b}_{t+1} + \Phi_{aq}\hat{a}_{t+1} + \Phi_{kq}M_{kk}\Phi_{kq}^{-1}\hat{q}_t \\
&\quad + \left[\Phi_{kq}M_{ka} - \Phi_{kq}M_{kk}\Phi_{kq}^{-1}\Phi_{aq}\right]\hat{a}_t + \left[\Phi_{kq}M_{kb} - \Phi_{kq}M_{kk}\Phi_{kq}^{-1}\Phi_{bq}\right]\hat{b}_t
\end{aligned}$$

Finally, take two adjacent periods, and use the definitions of $\omega_{t+1}^{A}\,(\equiv \hat{a}_{t+1} - \hat{a}_t)$ and

[40] Combine Equations 57 and 58:

$$\begin{aligned}
\hat{p}_t &= \mathbf{\Psi}_{21}^{-1}\hat{k}_{t+1} - \mathbf{\Psi}_{21}^{-1}\mathbf{\Psi}_{22}\hat{k}_t - \mathbf{\Psi}_{21}^{-1}\mathbf{\Phi}_2\begin{bmatrix}\hat{a}_t\\ \hat{b}_t\end{bmatrix} \\
&= \mathbf{\Psi}_{21}^{-1}\left[\mathbf{\Psi}_{22} - \mathbf{\Psi}_{21}(\mathbf{V}_{11}^{-1})^{-1}\mathbf{V}_{12}^{-1}\right]\hat{k}_t - \mathbf{\Psi}_{21}^{-1}\mathbf{\Psi}_{22}\hat{k}_t \\
&\quad -\mathbf{\Psi}_{21}^{-1}\mathbf{\Phi}_2\begin{bmatrix}\hat{a}_t\\ \hat{b}_t\end{bmatrix} + \mathbf{\Psi}_{21}^{-1}\left[\left(-\mathbf{\Psi}_{21}(\mathbf{V}_{11}^{-1})^{-1}\mathbf{D}_1^{-1}(\mathbf{I}-\mathbf{D}_1^{-1})^{-1}\tilde{\Phi}_1 + \mathbf{\Phi}_2\right)\right]\begin{bmatrix}\hat{a}_t\\ \hat{b}_t\end{bmatrix} \\
&= -(\mathbf{V}_{11}^{-1})^{-1}\mathbf{V}_{12}^{-1}\hat{k}_t - (\mathbf{V}_{11}^{-1})^{-1}\mathbf{D}_1^{-1}(\mathbf{I}-\mathbf{D}_1^{-1})^{-1}\tilde{\Phi}_1\begin{bmatrix}\hat{a}_t\\ \hat{b}_t\end{bmatrix}
\end{aligned}$$

So:

$$\begin{aligned}
\hat{q}_t = &\left\{(\mathbf{I}+\vartheta)\,\alpha - \left[(\vartheta + \varepsilon_Q\mathbf{I})\,(\mathbf{I}-S_M)^{-1}\,S_M\left(\mathbf{I}-S_1^M\right) + \vartheta\right](\mathbf{V}_{11}^{-1})^{-1}\mathbf{V}_{12}^{-1}\right\}\hat{k}_t \\
&+\left[\frac{\varepsilon_Q - 1}{\varepsilon_Q}\vartheta + \left(\frac{\vartheta}{\varepsilon_Q} + \mathbf{I}\right)(\mathbf{I}-S_M)^{-1}\,(\mathbf{I}+S_M\,(\varepsilon_Q - 1))\right]\hat{a}_t + (\mathbf{I}+\vartheta)\,(\mathbf{I}-\alpha)\,\hat{b}_t \\
&-\left[(\vartheta + \varepsilon_Q\mathbf{I})\,(\mathbf{I}-S_M)^{-1}\,S_M\left(\mathbf{I}-S_1^M\right) + \vartheta\right](\mathbf{V}_{11}^{-1})^{-1}\mathbf{D}_1^{-1}(\mathbf{I}-\mathbf{D}_1^{-1})^{-1}\tilde{\Phi}_1\begin{bmatrix}\hat{a}_t\\ \hat{b}_t\end{bmatrix}
\end{aligned}$$

$\omega^B_{t+1}\left(\equiv \hat{b}_{t+1}-\hat{b}_t\right)$ so that

$$
\begin{aligned}
\Delta\hat{q}_{t+1} &= \Phi_{kq}M_{kk}\Phi_{kq}^{-1}\Delta\hat{q}_t+\Phi_{bq}\omega^B_{t+1}+\Phi_{aq}\omega^A_{t+1}+\\
&\quad+\left[\Phi_{kq}M_{ka}-\Phi_{kq}M_{kk}\Phi_{kq}^{-1}\Phi_{aq}\right]\omega^A_t+\left[\Phi_{kq}M_{kb}-\Phi_{kq}M_{kk}\Phi_{kq}^{-1}\Phi_{bq}\right]\omega^B_t
\end{aligned}
$$

Parsing out the factor-neutral productivity shocks yields Equation 10 of the paper. This equation also describes how one can recover labor-augmenting productivity shocks using data on industries' output growth rates.

In the remainder of this subsection, we work out the expression for industries' value added growth rates. Begin with the first-order condition for industries' intermediate input purchases

$$
\begin{aligned}
M_{tJ}P^{in}_{tJ} &= \mu_J A_{tJ}^{\varepsilon_Q-1}\left(\frac{P^{in}_{tJ}}{P_{tJ}}\right)^{1-\varepsilon_Q}P_{tJ}Q_{tJ}\\
VA_{tJ} &= P_{tJ}Q_{tJ}-M_{tJ}P^{in}_{tJ}\\
&= P_{tJ}Q_{tJ}\left[1-\mu A_{tJ}^{\varepsilon_Q-1}\left(\frac{P^{in}_{tJ}}{P_{tJ}}\right)^{1-\varepsilon_Q}\right]\\
\frac{VA_{tJ}}{P_{tJ}} &= Q_{tJ}\cdot\left[1-\mu A_{tJ}^{\varepsilon_Q-1}\left(\frac{P^{in}_{tJ}}{P_{tJ}}\right)^{1-\varepsilon_Q}\right]
\end{aligned}
$$

So, the log-linearized expression for real value added is

$$
\hat{v}_t=\hat{q}_t-S^M\cdot(\varepsilon_Q-1)\cdot\hat{a}_t-S^M\cdot(\varepsilon_Q-1)\cdot\left(\mathbf{I}-S^M_1\right)\hat{p}_t
$$

Substituting out the expression for $\hat{q}_t$:

$$
\begin{aligned}
\hat{v}_t &= \Phi_{kq}\hat{k}_t+\Phi_{bq}\hat{b}_t+\left[\Phi_{aq}-S^M\cdot(\varepsilon_Q-1)\right]\hat{a}_t\\
&\quad-S^M\cdot(\varepsilon_Q-1)\cdot\left(\mathbf{I}-S^M_1\right)\hat{p}_t
\end{aligned}
$$

And then substituting out the expression for $\hat{p}_t$:

$$
\begin{aligned}
\Delta\hat{v}_t &= \left[\Phi_{kq}+S^M\cdot(\varepsilon_Q-1)\cdot\left(\mathbf{I}-S^M_1\right)\cdot(\mathbf{V}^{-1}_{11})^{-1}\mathbf{V}^{-1}_{12}\right]\Delta\hat{k}_t\\
&\quad+\Phi_{bq}\omega^B_t+\left[\Phi_{aq}-S^M\cdot(\varepsilon_Q-1)\right]\omega^A_t\\
&\quad+S^M\cdot(\varepsilon_Q-1)\cdot\left(\mathbf{I}-S^M_1\right)\cdot(\mathbf{V}^{-1}_{11})^{-1}\mathbf{D}^{-1}_1(\mathbf{I}-\mathbf{D}^{-1}_1)^{-1}\tilde{\mathbf{\Phi}}_1\begin{bmatrix}\omega^A_t\\ \omega^B_t\end{bmatrix}
\end{aligned}
$$

Equation 57 allows one to recursively compute the variance-covariance matrix of $\Delta\hat{k}_t$. From here, in combination with the last equation, one can write the covariance matrix of value added as a function of the covariance matrix of sectoral productivity shocks.

F.6 Calculations related to Section 2.4

In this section, I solve for covariance of industries' output as functions of the model parameters and the exogenous TFP terms. The solution involves three steps. First, I solve for the wage. Second, I solve for the relative prices and intermediate input cost shares. Third, I solve for real sales. As there is no capital or durable goods, the decisions within each period are independent of those made in other periods. As such, I will omit time subscripts in this section.

Step 1: For later use, I will first solve for the wage in each period. For this portion of the analysis, it will be sufficient to examine how much the consumer wants to work and how much she wants to consume. Since the consumer's problems are separable across periods, the objective function for the consumer is

$$\begin{aligned} \mathcal{U} &= \log C - \frac{\varepsilon_{LS}}{\varepsilon_{LS}+1} L^{\frac{\varepsilon_{LS}+1}{\varepsilon_{LS}}} \text{ subject to} \\ P \cdot C &= W \cdot L \ . \end{aligned}$$

The solution to this constrained optimization problem is:

$$W = L^{\frac{1}{\varepsilon_{LS}}} \text{ and } C = \frac{1}{P} \ . \tag{61}$$

Invoking the budget constraint of the representative consumer:

$$L^{\frac{\varepsilon_{LS}+1}{\varepsilon_{LS}}} = 1,$$

implying $W = 1$.

Step 2: Now consider the cost-minimization problem of the representative firm in industry J. As I argued in the text, the cost-minimization problem implies the following recursive equation for the marginal cost (equivalently, price) of industry J's good:

$$P_J = \frac{1}{A_J} \left[1 - \mu + \mu \left[\sum_{I=1}^{N} \frac{1}{N} \left(P_I\right)^{1-\varepsilon_M} \right]^{\frac{1-\varepsilon_Q}{1-\varepsilon_M}} \right]^{\frac{1}{1-\varepsilon_Q}} \quad \text{for } J = \{1, ...N\}. \tag{62}$$

The log-linear approximation to the previous equation is:

$$\log P_J \approx -\log A_J + \frac{\mu}{N} \sum_{I=1}^{N} \log P_I \ . \tag{63}$$

for all pairs of industries, so that Equation 63 implies:

$$\log P_J \approx -\log A_J + \frac{\mu}{N} \sum_{I=1}^{N} \left[\log P_J + \log A_J - \log A_I\right] \ .$$

Re-arranging:

$$\log P_J \approx -\log A_J - \frac{\mu}{N(1-\mu)} \sum_{I=1}^{N} \log A_I \ .$$

Because all industries' cost shares are identical (both in the consumer's preferences and in the production of each industry's intermediate input bundle):

$$\log P_J^{in} \approx \log P \approx -\frac{1}{N(1-\mu)} \sum_{J=1}^{N} \log A_J \ .$$

Step 3: The last task is to solve for Q_J. To do so, apply the market clearing condition for good I, plug in the intermediate input demand by customers of I, then re-arrange:

$$\begin{aligned}
Q_I &= C_I + \sum_{J=1}^{N} M_{I\to J} \ . \\
Q_I &= C_I + \frac{\mu}{N} (P_I)^{-\varepsilon_M} \sum_{J=1}^{N} Q_J (P_J)^{\varepsilon_Q - 1} \left(P_J^{in}\right)^{\varepsilon_M - \varepsilon_Q}
\end{aligned}$$

Next, take the log-linear approximation around the point at which all of the A's equal 1:

$$\begin{aligned}
\log Q_I &\approx \log\left(\frac{1}{1-\mu}\right) + (1-\mu)\log C_I - \mu\varepsilon_M \log P_I + \frac{\mu}{N}\sum_J \log Q_J \\
&\quad + \frac{\mu}{N}\sum_{J=1}^{N} (\varepsilon_Q - 1)\log P_J + (\varepsilon_M - \varepsilon_Q)\log P_J^{in} \ . \\
\log Q_I - \frac{\mu}{N}\sum_J \log Q_J &\approx \log\left(\frac{1}{1-\mu}\right) + (1-\mu)\log C_I - \mu\varepsilon_M \log P_I + \frac{\mu}{N}\sum_{J=1}^{N} (\varepsilon_M - 1)\log P_J \\
&\approx \log\left(\frac{1}{1-\mu}\right) + (1-\mu)\log C_I \qquad (64) \\
&\quad + \mu\varepsilon_M \log A_I + \frac{\mu\left[\varepsilon_M(\mu-1)+1\right]}{N(1-\mu)} \sum_{I=1}^{N} \log A_J
\end{aligned}$$

Given the preferences of the representative consumer, the demand function for good I is:

$$\begin{aligned}
\log C_I &= \log\frac{1}{N} - \varepsilon^D \log\left(\frac{P_I}{P}\right) - \log P \ . \\
&\approx \log\frac{1}{N} + \varepsilon^D \frac{1}{N}\sum_{J=1}^{N} \log\left(\frac{A_I}{A_J}\right) + \frac{1}{N(1-\mu)}\sum_{J=1}^{N} \log A_J \\
&\approx \log\frac{1}{N} + \varepsilon^D \log A_I + \frac{1-(1-\mu)\varepsilon_D}{N(1-\mu)} \sum_{J=1}^{N} \log A_J
\end{aligned}$$

Plug this expression back into Equation 64 and combine terms:

$$\begin{aligned}
\log Q_I - \frac{\mu}{N}\sum_J \log Q_J &\approx (1-\mu)\log\frac{1}{N} + \log\left(\frac{1}{1-\mu}\right) + (\mu\varepsilon_M + (1-\mu)\varepsilon_D)\log A_I \\
&\quad + \left[\frac{(1-\mu)(1-(1-\mu)\varepsilon_D) + \mu\left[\varepsilon_M(\mu-1)+1\right]}{N}\right]\sum_{J=1}^{N}\frac{\log A_J}{1-\mu} \\
&\approx (1-\mu)\log\frac{1}{N} + \log\left(\frac{1}{1-\mu}\right) + (\mu\varepsilon_M + (1-\mu)\varepsilon_D)\log A_I \\
&\quad + \frac{1-(1-\mu)(\mu\varepsilon_M + (1-\mu)\varepsilon_D)}{N}\sum_{J=1}^{N}\frac{\log A_J}{1-\mu} \qquad (65)
\end{aligned}$$

Equation 65 is a system of N linear equations. The solution to these equations are

$$\begin{aligned}
\log Q_I &\approx \log\frac{1}{N} + \frac{1}{1-\mu}\log\left(\frac{1}{1-\mu}\right) + (\mu\varepsilon_M + (1-\mu)\varepsilon_D)\log A_I \\
&\quad + \frac{1}{N}\left[\left(\frac{1}{1-\mu}\right)^2 - (\mu\varepsilon_M + (1-\mu)\varepsilon_D)\right]\sum_{J=1}^{N}\log A_J \qquad (66)
\end{aligned}$$

Equation 66 is equivalent to the expression given in the body of the paper.

F.7 Calculations related to Section 3

In this appendix, I demonstrate that the instrumental variable strategy outlined in Acemoglu, Akcigit, and Kerr (2016) extends to a set-up in which sectoral production functions are CES rather than Cobb-Douglas. To do so, I will extend the benchmark model to explicitly accommodate demand shocks. As in Acemoglu, Akcigit, and Kerr (2016), the model will be static, with neither capital nor durable consumption goods. Also as in Acemoglu, Akcigit, and Kerr (2016), I impose that the logarithm of productivity equals zero: $\log A_I = \log B_I = 0$ for all industries, I.

The goal of this exercise is to examine how a demand shock in one industry—in particular the Government industry, which would be directly affected by an exogenous increase in military spending—impacts output in other industries. In particular, I wish to show that a linear relationship exists irrespective of the values of ε_M and ε_Q.

Begin with the Lagrangian of the social planner's problem, dropping t subscripts:

$$\begin{aligned}
\mathcal{L} &= \sum_{I'}(D_{I'}\xi_{I'})^{\frac{1}{\varepsilon_D}}\cdot\log\left[\left[\sum_{J=1}^{N}(D_J\xi_J)^{\frac{1}{\varepsilon_D}}(C_{tJ})^{\frac{\varepsilon_D-1}{\varepsilon_D}}\right]^{\frac{\varepsilon_D}{\varepsilon_D-1}}\right] \\
&\quad - \frac{\varepsilon_{LS}}{\varepsilon_{LS}+1}\left(\sum_{J=1}^{N}L_J\right)^{\frac{\varepsilon_{LS}+1}{\varepsilon_{LS}}} + P_J\left[Q_J - C_J - \sum_{I=1}^{N}M_{J\to I}\right]
\end{aligned}$$

The production function is, as before:

$$
\begin{aligned}
Q_J &= A_J \cdot \left[(1-\mu_J)^{\frac{1}{\varepsilon_Q}} (L_J \cdot B_{tJ})^{\frac{\varepsilon_Q - 1}{\varepsilon_Q}} + (\mu_J)^{\frac{1}{\varepsilon_Q}} (M_J)^{\frac{\varepsilon_Q-1}{\varepsilon_Q}} \right]^{\frac{\varepsilon_Q}{\varepsilon_Q - 1}}, \text{ where} \\
M_J &\equiv \left[\sum_I \left(\Gamma_{IJ}^M\right)^{\frac{1}{\varepsilon_M}} (M_{IJ})^{\frac{\varepsilon_M - 1}{\varepsilon_M}} \right]^{\frac{\varepsilon_M}{\varepsilon_M - 1}}
\end{aligned}
$$

The first-order conditions associated with the planner's problem are:

$$
\begin{aligned}
P_J &= (D_J \xi_J)^{\frac{1}{\varepsilon_D}} (C_J)^{-\frac{1}{\varepsilon_D}} \left(\sum_{I=1}^{N} \frac{(\xi_I D_I)^{\frac{1}{\varepsilon_D}}}{\sum_{I'} (D_{I'} \xi_{I'})^{\frac{1}{\varepsilon_D}}} (C_I)^{\frac{\varepsilon_D - 1}{\varepsilon_D}} \right)^{-1} \qquad (67) \\
\frac{P_I}{P_J} &= (A_J)^{\frac{\varepsilon_Q-1}{\varepsilon_Q}} \left(\frac{Q_J \cdot \mu_J}{M_J} \right)^{\frac{1}{\varepsilon_Q}} \left(\frac{M_J \cdot \Gamma_{IJ}^M}{M_{I \to J}} \right)^{\frac{1}{\varepsilon_M}}. \\
\left(\sum_{J'=1}^{N} L_{J'} \right)^{\frac{1}{\varepsilon_{LS}}} &= P_J (A_J)^{\frac{\varepsilon_Q - 1}{\varepsilon_Q}} B_J (Q_J (1-\mu_J))^{\frac{1}{\varepsilon_Q}} (L_J \cdot B_J)^{-\frac{1}{\varepsilon_Q}}. \\
(1-\mu_J)^{\frac{1}{\varepsilon_Q}} (L_J \cdot B_J)^{\frac{\varepsilon_Q-1}{\varepsilon_Q}} &= (P_J)^{\varepsilon_Q - 1} (A_J)^{\frac{(\varepsilon_Q-1)^2}{\varepsilon_Q}} (B_J)^{\varepsilon_Q - 1} (Q_J)^{\frac{\varepsilon_Q - 1}{\varepsilon_Q}} (1-\mu_J) \left(\sum_{J'=1}^{N} L_{J'} \right)^{\frac{1-\varepsilon_Q}{\varepsilon_{LS}}} \\
\mu_J^{\frac{1}{\varepsilon_Q}} (M_J)^{\frac{\varepsilon_Q-1}{\varepsilon_Q}} &= (P_J)^{\varepsilon_Q - 1} (A_J)^{\frac{(\varepsilon_Q-1)^2}{\varepsilon_Q}} (Q_J)^{\frac{\varepsilon_Q-1}{\varepsilon_Q}} \mu_J \left(P_J^{in}\right)^{1-\varepsilon_Q}
\end{aligned}
$$

Combining the appropriate first-order conditions, and setting labor as the numeraire good (so that $\left(\sum_{J'=1}^{N} L_{J'}\right)^{\frac{1}{\varepsilon_{LS}}} = 1$) yields the following expression for industries' prices:

$$
P_J^{1-\varepsilon_Q} = A_J^{\varepsilon_Q - 1} \cdot \left[(1-\mu_J) B_J^{\varepsilon_Q - 1} + \mu_J \left(\sum_I \Gamma_{IJ}^M P_I^{1-\varepsilon_M} \right)^{\frac{1-\varepsilon_Q}{1-\varepsilon_M}} \right]
$$

Importantly, sectoral prices do not depend on the demand shocks. Also, with $\log A_I = \log B_I = 0$, all sectoral prices equal 1.

As a second step, manipulating Equation 67 and invoking the fact that sectoral prices are all equal 1, yields

$$
C_I = D_I \xi_I
$$

Plugging this expression into the market clearing condition

$$
Q_I = D_I \xi_I + \sum_J M_{I \to J}
$$

Since prices are constant, $d\log Q_I = \frac{d(Q_I P_I)}{Q_I P_I} = \frac{dQ_I}{Q_I}$ and

$$\begin{aligned} Q_I P_I &= D_I \xi_I P_I + \sum_J M_{I\to J} P_I \\ &= D_I \xi_I P_I + \sum_J P_J \Gamma_{IJ} Q_J \mu_J \end{aligned}$$

So

$$\begin{aligned} \frac{d\left(Q_I P_I\right)}{Q_I P_I} &= \xi_I \frac{dD_I}{Q_I} + \sum_J \Gamma_{IJ}\mu_J \frac{d\left(Q_J P_J\right)}{Q_I P_I} \\ &= \xi_I \frac{dD_I}{Q_I} + \sum_J \frac{\Gamma_{IJ}\mu_J Q_J P_J}{Q_I P_I}\frac{d\left(Q_J P_J\right)}{Q_J P_J} \end{aligned}$$

In matrix form, industries' output levels are given by:

$$\mathbf{d}\log\mathbf{Q} = \left(\mathbf{I} - \tilde{\mathbf{\Gamma}}\right)^{-1}\mathbf{dD}, \tag{68}$$

where the elements of $\tilde{\mathbf{\Gamma}}$ are given by $\frac{\Gamma_{IJ}\mu_J Q_J P_J}{Q_I P_I}$. Equation 68 is equivalent to Equation A8 from Acemoglu, Akcigit, and Kerr (2016).[41] Briefly, the reason why the result from Acemoglu, Akcigit, and Kerr (2016) extends to the current environment is that i) the impact of a demand shock on industries' sales depends on the production elasticities of substitution only if industries' prices react to demand shocks, but ii) demand shocks do not alter industries' prices.

F.8 Solution of the model filter with government demand shocks

In this subsection I work through a version of the model filter in which the government industry is not subject to productivity shocks. Instead, there are demand shocks in the government industry. For this robustness check, I assume that all goods are nondurable. I begin with the first-order condition from Equation 67

$$P_{tJ} = \left(D_{tJ}\xi_{tJ}\right)^{\frac{1}{\varepsilon_D}}\left(C_{tJ}\right)^{-\frac{1}{\varepsilon_D}}\left(\sum_{I=1}^{N}\frac{\left(D_{tI}\xi_I\right)^{\frac{1}{\varepsilon_D}}}{\sum_{I'}\left(D_{tI'}\xi_{I'}\right)^{\frac{1}{\varepsilon_D}}}\left(C_{tI}\right)^{\frac{\varepsilon_D-1}{\varepsilon_D}}\right)^{-1}$$

Notice that demand shocks do not appear in any of the other first-order conditions. Nor do they enter in the market-clearing conditions. To compute the log-linear approximation

[41] A parameter, λ, from Acemoglu, Akcigit, and Kerr (2016) describes the labor supply response from a change in government spending. The equation, here, is consistent with $\lambda \to \infty$.

(around the point at which all productivity and demand shocks equals 1), begin with

$$\frac{P_{tJ}}{P_J} \cdot P_J = (D_{tJ}\xi_J)^{\frac{1}{\varepsilon_D}} (C_J)^{-\frac{1}{\varepsilon_D}} (\exp\{\hat{c}_{tJ}\})^{-\frac{1}{\varepsilon_D}} \times \left(\sum_{I=1}^{N} \frac{(D_{tI}\xi_I)^{\frac{1}{\varepsilon_D}}}{\sum_{I'} (D_{tI'}\xi_{I'})^{\frac{1}{\varepsilon_D}}} \left(\frac{C_{tI}}{C_I}\right)^{\frac{\varepsilon_D - 1}{\varepsilon_D}} (C_I)^{\frac{\varepsilon_D-1}{\varepsilon_D}} \right)^{-1}$$

and substitute in the steady-state relationship between consumption and prices

$$\exp\{\hat{p}_{tJ}\} = \exp\left\{\hat{d}_{tJ}\right\}^{\frac{1}{\varepsilon_D}} \exp\{\hat{c}_{tJ}\}^{-\frac{1}{\varepsilon_D}} \times \left(\sum_{I=1}^{N} \frac{\exp\left\{\hat{d}_{tI}\right\}^{\frac{1}{\varepsilon_D}}}{\sum_{I'} \exp\left\{\hat{d}_{tI'}\right\}^{\frac{1}{\varepsilon_D}} (\xi_{I'})^{\frac{1}{\varepsilon_D}}} \frac{(\xi_I)^{\frac{1}{\varepsilon_D}} (C_I)^{\frac{\varepsilon_D-1}{\varepsilon_D}}}{\sum_{I'=1}^{N} (\xi_I')^{\frac{1}{\varepsilon_D}} (C_{I'})^{\frac{\varepsilon_D-1}{\varepsilon_D}}} \exp\{\hat{c}_{tI}\}^{\frac{\varepsilon_D-1}{\varepsilon_D}} \right)^{-1}$$

Taking derivatives of the logarithm of each side of the previous equation, around the point at which $\hat{p}_{tJ} = 0$, $\hat{d}_{tJ} = 0$, and $\hat{c}_{tJ} = 0$ yields:

$$\hat{p}_{tJ} = \frac{1}{\varepsilon_D}\hat{d}_{tJ} - \frac{1}{\varepsilon_D}\hat{c}_{tJ} + \sum_I \frac{(\xi_I)^{\frac{1}{\varepsilon_D}} (C_I)^{\frac{\varepsilon_D-1}{\varepsilon_D}}}{\sum_{I'=1}^{N} (\xi_I')^{\frac{1}{\varepsilon_D}} (C_{I'})^{\frac{\varepsilon_D-1}{\varepsilon_D}}} [1 - \varepsilon_D] \left[\hat{c}_{tI} - \hat{d}_{tI}\right]$$

In vector form, the log-linearized equation for consumption as a function of prices and demand shocks is:

$$\begin{aligned} \hat{p}_t &= \left[\mathbf{I} + S_I^C (\varepsilon_D - 1)\right] \left[-\frac{1}{\varepsilon_D}\hat{c}_t + \frac{1}{\varepsilon_D}\hat{d}_t\right] \Rightarrow \\ \hat{c}_t &= \hat{d}_t - \varepsilon_D \left[\mathbf{I} + S_I^C (\varepsilon_D - 1)\right]^{-1} \hat{p}_t \end{aligned}$$

Plug this log-linearized equation into Equation 52 to substitute out the $\hat{c}_t$ vector.

$$\begin{aligned} 0 = {} & \left[(\varepsilon_Q - 1)\tilde{S}_M^Q T_1 - \left(\mathbf{I} - \tilde{S}_M^Q T_1\right)\left(\mathbf{I} + \vartheta\varepsilon_Q^{-1}\right)(\mathbf{I} - S_M)^{-1}(\mathbf{I} + S_M(\varepsilon_Q - 1)) - \left(\mathbf{I} - \tilde{S}_M^Q T_1\right)\vartheta\frac{\varepsilon_Q - 1}{\varepsilon_Q}\right]\hat{a}_t \\ & + \tilde{S}_X^Q T_1 \delta_K^{-1}\hat{k}_{t+1} + \left[\tilde{S}_X^Q T_1\left(1 - \delta_K^{-1}\right) - \left(\mathbf{I} - \tilde{S}_M^Q T_1\right)(\mathbf{I} + \vartheta)\alpha\right]\hat{k}_t - \left(\mathbf{I} - \tilde{S}_M^Q T_1\right)(\mathbf{I} + \vartheta)(\mathbf{I} - \alpha)\hat{b}_t + \tilde{S}_C^Q \hat{d}_t \\ & + \left[-\varepsilon_D \tilde{S}_C^Q \left[\mathbf{I} + S_I^C(\varepsilon_D - 1)\right]^{-1} + \varepsilon_Q \tilde{S}_M^Q T_1\left(\mathbf{I} - S_1^M\right) + \varepsilon_M \tilde{S}_M^Q\left[T_1 S_1^M - T_2\right] + \varepsilon_X \tilde{S}_X^Q\left[T_1 S_1^X - T_2\right]\right]\hat{p}_t \\ & - \left(\mathbf{I} - \tilde{S}_M^Q T_1\right)\left[\varepsilon_Q(\mathbf{I} - S_M)^{-1} S_M\left(\mathbf{I} - S_1^M\right) + \vartheta\left[(\mathbf{I} - S_M)^{-1} S_M\left(\mathbf{I} - S_1^M\right) + \mathbf{I}\right]\right]\hat{p}_t \\ 0 = {} & -S_1^X \hat{p}_t + \left[\beta(1 - \delta_K)S_1^X + \tilde{\beta}(\mathbf{I} + \vartheta)\left(\mathbf{I} + (I - S_M)^{-1} S_M\left(\mathbf{I} - S_1^M\right)\right)\right]\hat{p}_{t+1} \\ & + \tilde{\beta}(\mathbf{I} + \vartheta)\left[\frac{1}{\varepsilon_Q}(I - S_M)^{-1}(\mathbf{I} + S_M(\varepsilon_Q - 1)) + \frac{\varepsilon_Q - 1}{\varepsilon_Q}\mathbf{I}\right]\hat{a}_{t+1} \\ & + \tilde{\beta}(\mathbf{I} + \vartheta)(\mathbf{I} - \alpha)\hat{b}_{t+1} + \tilde{\beta}(-\mathbf{I} + \alpha + \vartheta\alpha)\hat{k}_{t+1} \end{aligned}$$

Again, now with demand shocks, we have expressed the reduced system as

$$\begin{bmatrix} \mathbb{E}_t[\hat{p}_{t+1}] \\ \hat{k}_{t+1} \end{bmatrix} = \mathbf{\Psi} \begin{bmatrix} \hat{p}_t \\ \hat{k}_t \end{bmatrix} + \overset{d}{\mathbf{\Phi}} \begin{bmatrix} \hat{a}_t \\ \hat{b}_t \\ \hat{d}_t \end{bmatrix}$$

As in Appendix F.4, $\mathbf{\Psi}$ has N stable and N unstable eigenvalues. Here, the "d" in $\overset{d}{\mathbf{\Phi}}$ refers to the modification of Φ to allow for demand shocks. Using a similar set of calculations as in Appendix F.4, we arrive at the following equation for the evolution of the endogenous state:

$$\begin{aligned} \hat{k}_{t+1} \quad &= \quad \underbrace{(\mathbf{\Psi}_{22} - \mathbf{\Psi}_{21}(\mathbf{V}_{11}^{-1})^{-1}\mathbf{V}_{12}^{-1})}_{\equiv M_{kk}}\hat{k}_t \\ &\quad + \underbrace{\left(-\mathbf{\Psi}_{21}(\mathbf{V}_{11}^{-1})^{-1}\mathbf{D}_1^{-1}(\mathbf{I} - \mathbf{D}_1^{-1})^{-1}\overset{d}{\tilde{\Phi}}_1 + \overset{d}{\mathbf{\Phi}}_2 \right)}_{\equiv \left[\overset{d}{\mathbf{M}}_{ka},\, \overset{d}{\mathbf{M}}_{kb},\, \overset{d}{\mathbf{M}}_{kd} \right]} \begin{bmatrix} \hat{a}_t \\ \hat{b}_t \\ \hat{d}_t \end{bmatrix} \end{aligned} \tag{69}$$

As before

$$\hat{p}_t = \mathbf{\Psi}_{21}^{-1}\hat{k}_{t+1} - \mathbf{\Psi}_{21}^{-1}\mathbf{\Psi}_{22}\hat{k}_t - \mathbf{\Psi}_{21}^{-1}\overset{d}{\mathbf{\Phi}}_2 \begin{bmatrix} \hat{a}_t \\ \hat{b}_t \\ \hat{d}_t \end{bmatrix} \tag{70}$$

As before, the following equation describes $\hat{q}$ as a function of the exogenous variables, $\hat{k}$, and $\hat{p}$:

$$\begin{aligned} \hat{q}_t &= (\mathbf{I} + \vartheta)\,(\mathbf{I} - \alpha)\,\hat{b}_t + (\mathbf{I} + \vartheta)\,\alpha\hat{k}_t \\ &\quad + \left[\frac{\varepsilon_Q - 1}{\varepsilon_Q}\vartheta + \left(\frac{\vartheta}{\varepsilon_Q} + \mathbf{I} \right) (\mathbf{I} - S_M)^{-1} \left(\mathbf{I} + S_M\,(\varepsilon_Q - 1)\right) \right] \hat{a}_t \\ &\quad + \left[(\vartheta + \varepsilon_Q \mathbf{I})\,(\mathbf{I} - S_M)^{-1}\, S_M \left(\mathbf{I} - S_1^M\right) + \vartheta \right] \hat{p}_t \end{aligned} \tag{71}$$

Plug Equation 69 and 70 in to 71 so that we may write:

$$\hat{q}_t = \Phi_{kq}\hat{k}_t + \Phi_{bq}\hat{b}_t + \Phi_{aq}\hat{a}_t + \Phi_{dq}\hat{d}_t, \tag{72}$$

where the Φ_{kq}, Φ_{bq},Φ_{aq}, and Φ_{dq} are matrices that collect the appropriate terms.[42]

So long as Φ_{kq} is invertible, Equation 72 is equivalent to

$$\hat{k}_t = \Phi_{kq}^{-1}\hat{q}_t - \Phi_{kq}^{-1}\Phi_{bq}\hat{b}_t - \Phi_{kq}^{-1}\Phi_{aq}\hat{a}_t - \Phi_{kq}^{-1}\Phi_{dq}\hat{d}_t$$

Equation 72, one period ahead, is

$$\hat{q}_{t+1} = \Phi_{kq}\hat{k}_{t+1} + \Phi_{bq}\hat{b}_{t+1} + \Phi_{aq}\hat{a}_{t+1} + \Phi_{dq}\hat{d}_{t+1}$$

Apply Equation 69 to this previous equation

$$\begin{aligned}
\hat{q}_{t+1} &= \Phi_{bq}\hat{b}_{t+1} + \Phi_{aq}\hat{a}_{t+1} + \Phi_{dq}\hat{d}_{t+1} \\
&\quad +\Phi_{kq}\left(M_{kk}\hat{k}_t + \overset{d}{\mathbf{M}}_{ka}\hat{a}_t + \overset{d}{\mathbf{M}}_{kb}\hat{b}_t + \overset{d}{\mathbf{M}}_{kd}\hat{d}_t\right) \\
&= \Phi_{bq}\hat{b}_{t+1} + \Phi_{aq}\hat{a}_{t+1} + \Phi_{dq}\hat{d}_{t+1} \\
&\quad +\Phi_{kq}\overset{d}{\mathbf{M}}_{ka}\hat{a}_t + \Phi_{kq}\overset{d}{\mathbf{M}}_{kb}\hat{b}_t + \Phi_{kq}M_{kd}\hat{d}_t + \Phi_{kq}M_{kk}\Phi_{kq}^{-1}\hat{q}_t \\
&\quad -\Phi_{kq}M_{kk}\Phi_{kq}^{-1}\Phi_{bq}\hat{b}_t - \Phi_{kq}M_{kk}\Phi_{kq}^{-1}\Phi_{aq}\hat{a}_t - \Phi_{kq}M_{kk}\Phi_{kq}^{-1}\Phi_{dq}\hat{d}_t \\
&= \Phi_{dq}\hat{d}_{t+1} + \Phi_{bq}\hat{b}_{t+1} + \Phi_{aq}\hat{a}_{t+1} + \Phi_{kq}M_{kk}\Phi_{kq}^{-1}\hat{q}_t + \left[\Phi_{kq}\overset{d}{\mathbf{M}}_{ka} - \Phi_{kq}M_{kk}\Phi_{kq}^{-1}\Phi_{aq}\right]\hat{a}_t \\
&\quad + \left[\Phi_{kq}\overset{d}{\mathbf{M}}_{kb} - \Phi_{kq}M_{kk}\Phi_{kq}^{-1}\Phi_{bq}\right]\hat{b}_t + \left[\Phi_{kq}\overset{d}{\mathbf{M}}_{kd} - \Phi_{kq}M_{kk}\Phi_{kq}^{-1}\Phi_{dq}\right]\hat{d}_t
\end{aligned}$$

[42] Combine Equations 69 and 70:

$$\begin{aligned}
\hat{p}_t &= \mathbf{\Psi}_{21}^{-1}\hat{k}_{t+1} - \mathbf{\Psi}_{21}^{-1}\mathbf{\Psi}_{22}\hat{k}_t - \mathbf{\Psi}_{21}^{-1}\overset{d}{\mathbf{\Phi}}_2\begin{bmatrix}\hat{a}_t\\ \hat{b}_t\\ \hat{d}_t\end{bmatrix} \\
&= \mathbf{\Psi}_{21}^{-1}\left[\mathbf{\Psi}_{22} - \mathbf{\Psi}_{21}(\mathbf{V}_{11}^{-1})^{-1}\mathbf{V}_{12}^{-1}\right]\hat{k}_t - \mathbf{\Psi}_{21}^{-1}\mathbf{\Psi}_{22}\hat{k}_t \\
&\quad -\mathbf{\Psi}_{21}^{-1}\overset{d}{\mathbf{\Phi}}_2\begin{bmatrix}\hat{a}_t\\ \hat{b}_t\\ \hat{d}_t\end{bmatrix} + \mathbf{\Psi}_{21}^{-1}\left[\left(-\mathbf{\Psi}_{21}(\mathbf{V}_{11}^{-1})^{-1}\mathbf{D}_1^{-1}(\mathbf{I}-\mathbf{D}_1^{-1})^{-1}\overset{d}{\tilde{\Phi}}_1 + \overset{d}{\mathbf{\Phi}}_2\right)\right]\begin{bmatrix}\hat{a}_t\\ \hat{b}_t\\ \hat{d}_t\end{bmatrix} \\
&= -(\mathbf{V}_{11}^{-1})^{-1}\mathbf{V}_{12}^{-1}\hat{k}_t - (\mathbf{V}_{11}^{-1})^{-1}\mathbf{D}_1^{-1}(\mathbf{I}-\mathbf{D}_1^{-1})^{-1}\overset{d}{\tilde{\Phi}}_1\begin{bmatrix}\hat{a}_t\\ \hat{b}_t\\ \hat{d}_t\end{bmatrix}
\end{aligned}$$

So:

$$\begin{aligned}
\hat{q}_t = &\left\{(\mathbf{I}+\vartheta)\,\alpha - \left[(\vartheta+\varepsilon_Q\mathbf{I})\,(\mathbf{I}-S_M)^{-1}\,S_M\left(\mathbf{I}-S_1^M\right)+\vartheta\right](\mathbf{V}_{11}^{-1})^{-1}\mathbf{V}_{12}^{-1}\right\}\hat{k}_t \\
&+\left[\frac{\varepsilon_Q-1}{\varepsilon_Q}\vartheta + \left(\frac{\vartheta}{\varepsilon_Q}+\mathbf{I}\right)(\mathbf{I}-S_M)^{-1}\left(\mathbf{I}+S_M\left(\varepsilon_Q-1\right)\right)\right]\hat{a}_t + (\mathbf{I}+\vartheta)\,(\mathbf{I}-\alpha)\,\hat{b}_t \\
&-\left[(\vartheta+\varepsilon_Q\mathbf{I})\,(\mathbf{I}-S_M)^{-1}\,S_M\left(\mathbf{I}-S_1^M\right)+\vartheta\right](\mathbf{V}_{11}^{-1})^{-1}\mathbf{D}_1^{-1}(\mathbf{I}-\mathbf{D}_1^{-1})^{-1}\overset{d}{\tilde{\Phi}}_1\begin{bmatrix}\hat{a}_t\\ \hat{b}_t\\ \hat{d}_t\end{bmatrix}
\end{aligned}$$

Finally, take two adjacent periods, and use the definitions of ω^A_{t+1} $(\equiv \hat{a}_{t+1} - \hat{a}_t)$, ω^B_{t+1} $\left(\equiv \hat{b}_{t+1} - \hat{b}_t\right)$, and ω^D_{t+1} $\left(\equiv \hat{d}_{t+1} - \hat{d}_t\right)$ so that

$$\begin{aligned}
\Delta\hat{q}_{t+1} &= \Phi_{kq} M_{kk} \Phi^{-1}_{kq} \Delta\hat{q}_t + \Phi_{dq}\omega^D_{t+1} + \Phi_{bq}\omega^B_{t+1} + \Phi_{aq}\omega^A_{t+1} + \\
&\quad + \left[\Phi_{kq} \overset{d}{\mathbf{M}}_{ka} - \Phi_{kq} M_{kk} \Phi^{-1}_{kq} \Phi_{aq}\right] \omega^A_t + \left[\Phi_{kq} \overset{d}{\mathbf{M}}_{kb} - \Phi_{kq} M_{kk} \Phi^{-1}_{kq} \Phi_{bq}\right] \omega^B_t \\
&\quad + \left[\Phi_{kq} \overset{d}{\mathbf{M}}_{kd} - \Phi_{kq} M_{kk} \Phi^{-1}_{kq} \Phi_{dq}\right] \omega^D_t
\end{aligned}$$

In the robustness check with productivity shocks in all non-government industries in combination with demand shocks in the government sector, the filter is given by inverting the following equation:

$$\begin{aligned}
\Delta\hat{q}_{t+1} &= \Phi_{kq} M_{kk} \Phi^{-1}_{kq} \Delta\hat{q}_t \\
&\quad + \left[\left[\Phi_{aq}\right]_{[1:N,1:N-1]} \vdots\, \Phi_{dq[1:N,N]} \right] \cdot \begin{bmatrix} \left[\omega^A_{t+1}\right]_{[1:N-1]} \\ \left[\omega^D_{t+1}\right]_N \end{bmatrix} \\
&\quad + \left[\left[\Phi_{kq} \overset{d}{\mathbf{M}}_{ka} - \Phi_{kq} M_{kk} \Phi^{-1}_{kq} \Phi_{aq}\right]_{[1:N,1:N-1]} \vdots \left[\Phi_{kq} \overset{d}{\mathbf{M}}_{kd} - \Phi_{kq} M_{kk} \Phi^{-1}_{kq} \Phi_{dq}\right]_{[1:N,N]} \right] \\
&\quad \cdot \begin{bmatrix} \left[\omega^A_t\right]_{[1:N-1]} \\ \left[\omega^D_t\right]_N \end{bmatrix}
\end{aligned} \tag{73}$$

Since the government sector is the final (N^{th}) industry, the filter recovers an $N-1$ dimensional productivity vector along with a single, final element of the demand shock vector. In Equation 73, a $[1 : N-1]$ subscript refers to the first $N-1$ elements of a vector; a $[1 : N, 1 : N-1]$ subscript refers to the first $N-1$ columns of a given matrix; and a $[1 : N, N]$ subscript refers to the final column. Here, we have removed labor-augmenting productivity shocks and factor-neutral productivity shocks in the N^{th} (governmental) sector as a source of output fluctuations.

Additional references

Acemoglu, Daron, and Veronica Guerrieri. 2008. "Capital Deepening and Nonbalanced Economic Growth." *Journal of Political Economy*, 116(3): 467-498.

Atalay, Enghin. 2014. "Materials Prices and Productivity." *Journal of the European Economic Association*, 12(3): 575-611.

Canova, Fabio. 2014. "Bridging DSGE Models and the Raw Data." *Journal of Monetary Economics* 67(1): 1-15.

Foster, Lucia, John Haltiwanger, and Chad Syverson. 2016. "The Slow Growth of New

Plants: Learning about Demand?" *Economica*, 83(329): 91-129.

Hulten, Charles R., and Frank C. Wykoff. 1981. "The Measurement of Economic Depreciation." In *Depreciation, Inflation, and the Taxation of Income from Capital*, edited by Charles Hulten, Washington, DC: Urban Institute.

O' Mahony, Mary, and Marcel P. Timmer. 2009. "Output, Input and Productivity Measures at the Industry Level: The EU KLEMS Database." *Economic Journal*, 119(538): 374-403.

Timmer, Marcel, Ton van Moergastel, Edwin Stuivenwold, Gerard Ypma, Mary O' Mahony, and Mari Kangasniemi. 2007. "EU KLEMS Growth and Productivity Accounts Version 1.0." mimeo.